

The flavor of the composite pseudo-goldstone Higgs

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ABSTRACT: We study the flavor structure of 5D warped models that provide a dual description of a composite pseudo-Goldstone Higgs. We first carefully re-examine the flavor constraints on the mass scale of new physics in the standard Randall-Sundrum-type scenarios, and find that the KK gluon mass should generically be heavier than about 21 TeV. We then compare the flavor structure of the composite Higgs models to those in the RS model. We find new contributions to flavor violation, which while still are suppressed by the RS-GIM mechanism, will enhance the amplitudes of flavor violations. In particular, there is a kinetic mixing term among the SM fields which (although parametrically not enhanced) will make the flavor bounds even more stringent than in RS. This together with the fact that in the pseudo-Goldstone scenario Yukawa couplings are set by a gauge coupling implies the KK gluon mass to be at least about 33 TeV. For both the RS and the composite Higgs models the flavor bounds could be stronger or weaker depending on the assumption on the value of the gluon boundary kinetic term. These strong bounds seem to imply that the fully anarchic approach to flavor in warped extra dimensions is implausible, and there have to be at least some partial flavor symmetries appearing that eliminate part of the sources for flavor violation. We also present complete expressions for the radiatively generated Higgs potential of various 5D implementations of the composite Higgs model, and comment on the 1 – 5 percent level tuning needed in the top sector to achieve a phenomenologically acceptable vacuum state.

KEYWORDS: Beyond Standard Model, Quark Masses and SM Parameters, Field Theories in Higher Dimensions, Kaon Physics.

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1. Introduction

Warped extra dimensions models were introduced by Randall and Sundrum (RS) [1] as an attempt to solve the hierarchy problem by making use of the warp factor to lower the natural scale of particle masses. In the original model, all SM fields were localized on the TeV brane. By the AdS/CFT duality, this corresponds to a situation where a strongly coupled 4D conformal field theory spontaneously breaks conformality at the TeV scale, creates a mass gap (confines), and produces the SM fields as approximately massless composites. One consequence of this scenario is the CFT cannot be flavor invariant, since it is supposed to produce the Yukawa couplings among the SM fields. In such a case, however,

the CFT is also expected to generate higher-dimensional flavor violating operators with only a TeV suppression, which would be disastrous from the phenomenological point of view. Phrased in the 5D language this question is why generic TeV localized four-fermion operators are suppressed by some high scale, rather than the local cut-off scale which is a few TeV.

This severe flavor problem can be avoided by instead considering setups with the SM gauge fields and fermions in the bulk, and only the Higgs sharply localized on the TeV brane [2–4]. In this case the SM fermions can be thought of as mixtures of elementary and composite fermions. The amount of mixing is determined by the profile of the 5D wave function of the fermions: the more peaked they are close to the Planck brane, the more elementary are these fields. This sheds some light on the flavor puzzle as well: Although the Yukawa couplings generated by the CFT are indeed all $\mathcal{O}(1)$ and non-diagonal, the fermion masses and the CKM angles depend also on the amount of mixing of the elementary fermions with the CFT that is assumed to be small for the first two generations [3–5]. Most importantly, this implies that flavor violation in the SM is also suppressed by the same mixing factors - the fact that goes under the name of the RS-GIM mechanism [4, 6, 7], see also [8, 9]. RS-GIM is successful in suppressing most of the dangerous flavor-changing neutral currents (FCNC) [6], although it is not enough to sufficiently suppress new physics contributions to CP violation in the kaon sector [11, 10].

Although the RS set-up explains the origin of the large M_{PL}/TeV hierarchy, there still remains the little hierarchy problem, which amounts to the question why the Higgs boson is much lighter than a few TeV. In RS, the Higgs is realized as a scalar field localized on the TeV brane, of which the 4D dual interpretation is that Higgs a composite state of the CFT. In that case, its mass would naturally be at the CFT scale of a few TeV, which would effectively produce a Higgsless model of the sort considered in [12]. One way to obtain a light composite Higgs is by making it a pseudo-Goldstone boson (pGB) of a global symmetry [13], similarly as in little Higgs models [14]. The 5D holographic version of this scenario are the models of gauge-Higgs unification (GHU) [15], where the Higgs boson is identified with the fifth component of a bulk gauge field (A_5) [16]. The Higgs potential is radiatively generated (with the largest contributions due to the top and gauge multiplets) and fully calculable. The most promising scenario of this kind is based on the $\text{SO}(5)$ gauge group in the bulk, broken spontaneously to $\text{SO}(4)$ on the TeV brane [17], and with an implementation of a discrete parity symmetry to control corrections to the $Zb\bar{b}$ vertex [18]. This is the minimal scenario that passes the stringent electroweak precision tests.

Thus, GHU provides us with the Higgs sector that allows one to address both the large and the little hierarchy problem. It is a natural question to ask how this dynamical realization of the Higgs boson affects the flavor structure and the RS-GIM mechanism. This is the subject of this paper.

Perhaps not surprisingly, the flavor structure of GHU turns out to be quite similar to that in RS. However, there are some important differences, that affect the flavor bounds. The gauge symmetry must be larger than in RS (to include broken generators that give rise to the pseudo-Goldstone Higgs degrees of freedom), which results in a different embedding

of the SM fermion into the bulk representation. In particular, one SM fermion must be embedded in several bulk multiplets. This induces a new effect not present in RS, namely that, in the original flavor basis, the various fermionic generations are mixed via kinetic terms. This kinetic mixing is a new source of flavor violation in GHU which is always non-zero (as long as the CKM mixing is reproduced). Even though the kinetic mixing respects the RS-GIM mechanism, it results in an enhancement of the flavor violating operators. As a consequence, the bounds on the scale of the extra dimensions from flavor physics turn out to be more stringent compared to the RS case.

This paper is organized as follows: in section 2 we review the flavor structure and the RS-GIM mechanism of ordinary RS models with gauge and fermions in the bulk and the Higgs on the TeV brane. We reevaluate the bounds on the mass of the KK gluon and find it has to be ≥ 22 TeV for anarchic Yukawa couplings. In section 3 we review the basics of the GHU models and then define three different realizations based on the SO(5) gauge symmetry in the bulk. Before investigating the flavor structure, we study the constraints on the parameter space of these models imposed by the requirement of correct electroweak symmetry breaking. In section 4 we calculate the Higgs potential for the three SO(5) models. We re-emphasize that one of the parameters of the top sector has to be tuned at the 1-5 percent level to end up with a realistic scenario, which is a new guise of the little hierarchy problem specified to GHU models. These relations among the parameters of the model are then used as an input in our studies of the flavor bounds. Section 5 contains our main results. There we present the flavor structure of the SO(5) models under investigation, and pinpoint new sources of flavor violation. We estimate the magnitude of flavor and CP violation induced by the KK gluon exchange and illustrate our analytical estimates by numerical scans over the parameter space allowed by electroweak symmetry breaking. We conclude with some remarks on future directions of GHU in light of our findings.

2. Flavor in RS

The original motivation for considering warped extra dimension was the solution to the hierarchy problem of the Higgs sector. It was quickly realized that the set-up also has the potential to explain simultaneously the SM flavor structure. Starting with completely anarchic Yukawa coupling of the Higgs and the 5D fermions, the large SM fermion mass hierarchies can be explained by different localization the SM fermions in the extra dimension [3, 4, 19], implementing the split fermion scenario of [20]. Small mixing angles of the CKM matrix are a natural consequence of this scenario [5]. Moreover, this way of generating flavor mass hierarchies automatically implies a certain amount of suppression of the dangerous flavor changing, which is referred to as the RS-GIM mechanism. In this section we will review the flavor bounds on the generic RS models with anarchic flavor structure. This will provide us with a reference point for our study of the flavor bounds in the GHU models.

We specify the background metric to be AdS₅ space. We parametrize the space-time

by the conformal coordinates

$$ds^2 = \left(\frac{R}{z}\right)^2 (dx_\mu dx_\nu \eta^{\mu\nu} - dz^2), \quad (2.1)$$

where the AdS curvature is R , and the coordinate z of the extra dimension runs between $R < z < R'$, $z = R$ corresponding to the UV (Planck) brane and $z = R'$ to the IR (TeV) brane. $R'/R \sim 10^{16}$ sets the large hierarchy between the Planck and the TeV scale.

We consider here the standard RS scenario with custodial symmetry [21] (this is always what we mean when we refer to RS in the following). The bulk gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X$ and the Higgs field transforming as $(1, 2, 2)_0$ is localized on the TeV brane. The fermionic content includes three copies of Ψ_q^i , $i = 1 \dots 3$, transforming as $(2, 1)_{1/6}$, and 3 copies each of $\Psi_{u,d}^i$ in $(1, 2)_{1/6}$. Each of these fields are 5D bulk Dirac spinors and have a bulk mass term which is customarily parametrized by using the c -parameters,

$$\left(\frac{R}{z}\right)^4 \left[\frac{c_q^i}{z} \bar{\Psi}_q^i \Psi_q^i + \frac{c_u^i}{z} \bar{\Psi}_u^i \Psi_u^i + \frac{c_d^i}{z} \bar{\Psi}_d^i \Psi_d^i \right]. \quad (2.2)$$

From now on we drop the generation index i ; all fermions should always be understood as three-vectors in the generation space. The boundary conditions on the UV and the IR brane are chosen as

$$\Psi_q = \left(q[+, +] \right) \quad \Psi_u = \begin{pmatrix} u^c[-, -] \\ \tilde{d}^c[+, -] \end{pmatrix} \quad \Psi_d = \begin{pmatrix} \tilde{u}^c[+, -] \\ d^c[-, -] \end{pmatrix} \quad (2.3)$$

where $[\pm]$ denotes the right(left)-chirality of a bulk fermion vanishing on the brane. The SM quark doublets are realized as zero-modes q , while the singlets up and down-type quarks are zero modes of u^c and d^c , respectively. We write it as

$$q(x, z) \rightarrow \chi_q(z) q_L(x) \quad u^c(x, z) \rightarrow \psi_u(z) u_R(x) \quad d^c(x, z) \rightarrow \psi_d(z) d_R(x) \quad (2.4)$$

where $\chi_q(z)$ and $\psi_{u,d}(z)$ are the zero mode profiles that are obtained by solving the equations of motion. A normalized left-handed zero-mode profile is given by

$$\chi_c(z) = R'^{-1/2} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c} f(c), \quad \psi_c(z) = R'^{-1/2} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^c f(-c). \quad (2.5)$$

We have introduced the standard RS flavor function $f(c)$, which is given by

$$f(c) = \frac{\sqrt{1-2c}}{\left[1 - \left(\frac{R'}{R}\right)^{2c-1}\right]^{\frac{1}{2}}}. \quad (2.6)$$

We also introduce 3×3 diagonal matrices f_c that are constructed from $f(c_i)$ of three generations, for example $f_q = \text{diag}(f(c_{q_1}), f(c_{q_2}), f(c_{q_3}))$. For the choice of the c -parameters that reproduce the SM mass hierarchies the matrices $f_{q,-u,-d}$ are exponentially hierarchical.

The masses of the zero modes come from the IR brane-localized Yukawa interactions. After the Higgs field acquires a vev, the Yukawa terms lead to IR localized mass terms for the bulk fermions,

$$\mathcal{L}_y = -\frac{v}{\sqrt{2}} (R^4/R'^3) \left(\bar{\Psi}_q \tilde{Y}_u \Psi_u + \bar{\Psi}_q \tilde{Y}_d \Psi_d \right) + \text{h.c.} \quad (2.7)$$

	value at 3 TeV
\bar{m}_u	0.00075 ... 0.0015
\bar{m}_c	0.56 ± 0.04
\bar{m}_t	136.2 ± 3.1
\bar{m}_d	0.002 ... 0.004
\bar{m}_s	0.047 ± 0.012
\bar{m}_b	2.4 ± 0.04

Table 1: \overline{MS} quark masses in GeV at 3 TeV. We have taken the ranges and low-energy values from PDGLive [22] and used LO renormalization equations with the appropriate number of flavors for the rescaling. At 30 TeV, the masses \bar{m}_i are about 11% smaller.

The brane Yukawa couplings $\tilde{Y}_{u,d}$ are assumed to be anarchic — random matrices with elements $\mathcal{O}(1)$, no hierarchy and $\mathcal{O}(1)$ determinant.

Inserting the zero mode profiles into the mass terms in eq. (2.7) we obtain the SM mass matrices

$$\begin{aligned}
 m_u^{\text{SM}} &= \frac{v}{\sqrt{2}} f_q \tilde{Y}_u f_{-u}, \\
 m_d^{\text{SM}} &= \frac{v}{\sqrt{2}} f_q \tilde{Y}_d f_{-d},
 \end{aligned}
 \tag{2.8}$$

From this point on the usual SM prescription applies. We diagonalize the up and down mass matrices by $m_{u,d}^{\text{SM}} = U_{L\ u,d} m_{u,d} U_{R\ u,d}^\dagger$, where U 's are unitary and $m_{u,d}$ are diagonal, and we rotate the zero modes to the mass eigenstate basis, for example $d_L(x) \rightarrow U_{L\ d} d_L(x)$. The left rotations yield the CKM matrix, $V_{\text{CKM}} = U_{L\ u}^\dagger U_{L\ d}$.

Even though the Yukawa matrices are anarchical, the hierarchical matrices f_c introduce the hierarchy into the mass matrix elements.

This will result in the hierarchy of the eigenvalues:¹

$$(m_{u,d})_{ii} \sim \frac{v}{\sqrt{2}} Y_* f_{q_i} f_{-u_i, d_i}
 \tag{2.9}$$

where Y_* is the typical amplitude of the entries in the Yukawa matrices. One can also show that the diagonalization matrices themselves are hierarchical [5]:

$$|U_{L\ ij}| \sim \frac{f_{q_i}}{f_{q_j}}, \quad |U_{R\ ij}| \sim \frac{f_{-u_i, d_i}}{f_{-u_j, d_j}}, \quad i \leq j.
 \tag{2.10}$$

We also get that $|V_{\text{CKM}}|_{ij} \sim f_{q_i}/f_{q_j}$, thus the hierarchy in the CKM matrix elements is purely set by the c_q parameters. From experiment we know that the hierarchy of the CKM matrix is of the form

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & \lambda^3 \\ \lambda & 1 - \frac{\lambda^2}{2} & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}
 \tag{2.11}$$

¹Here and in the following, the quark masses are understood as running masses at the scale at which the extra dimension is integrated out. We choose the scale of the extra dimension to be 3 TeV. In table 1 we collect all input values at this scale.

where $\lambda \sim \sin \theta_C \sim 0.2$. This fixes the hierarchy among the f_{q_i} 's to be [5]

$$f_{q_2}/f_{q_3} \sim \lambda^2, \quad f_{q_1}/f_{q_3} \sim \lambda^3. \quad (2.12)$$

The values of $f_{-u,-d}$ are then fixed by requiring that the correct fermion mass hierarchy is reproduced, implying the following relations (assuming $f_{-u_3} \sim \mathcal{O}(1)$):

$$f_{-d_3} \sim \frac{m_b}{m_t}, \quad f_{-u_2} \sim \frac{m_c}{m_t} \frac{1}{\lambda^2}, \quad f_{-d_2} \sim \frac{m_s}{m_t} \frac{1}{\lambda^2}, \quad f_{-u_1} \sim \frac{m_u}{m_t} \frac{1}{\lambda^3}, \quad f_{-d_1} \sim \frac{m_d}{m_t} \frac{1}{\lambda^3}. \quad (2.13)$$

Thus, the RS set-up leads to a neat explanation of the SM flavor structure. However, one potentially worrisome feature of higher-dimensional models is the presence of the new KK states, whose masses are in the TeV range (as long as the hierarchy problem is addressed). These new states generically have flavor non-universal couplings and will contribute to flavor-changing neutral currents.

The largest contribution to flavor changing neutral currents is generated via the exchange of heavy gauge bosons, in particular the strongest constraint arises from the exchange of the KK gluons. In order to calculate the effective four-Fermi operators we first need to determine the couplings g_x of the zero-modes to the KK gluons,

$$g_{L,u}^{ij} \bar{u}_L^i \gamma_\mu G^{\mu(1)} u_L^j + g_{L,d}^{ij} \bar{d}_L^i \gamma_\mu G^{\mu(1)} d_L^j + (L \rightarrow R) \quad (2.14)$$

Below we discuss the contribution of the lightest KK gluon but, as we show in appendix A, it is possible to sum up the contribution of the entire gluon KK tower. The profile of the first KK gluon can be approximated by $G^{(1)}(z) \simeq \frac{\sqrt{2}}{J_1(x_1)\sqrt{RR'}} z J_1(x_1 z/R')$ with x_1 being the first zero of the Bessel function, $J_0(x_1) = 0$. Using this and the zero mode profiles we can determine the couplings in eq. (2.14). In the original flavor basis the couplings are diagonal and well approximated by

$$g_x \approx g_{s*} \left(-\frac{1}{\log R'/R} + f_x^2 \gamma(c_x) \right). \quad (2.15)$$

where g_{s*} is the bulk SU(3) gauge couplings, and $\gamma(c) = \frac{\sqrt{2}x_1}{J_1(x_1)} \int_0^1 x^{1-2c} J_1(x_1 x) dx \approx \frac{\sqrt{2}x_1}{J_1(x_1)} \frac{0.7}{6-4c}$.² The couplings would be flavor universal if $f_x \sim 1_{3 \times 3}$. However, this is not the case in the RS scenario where the f_x are non-degenerate. This is the main source of flavor violation in RS. Going to the mass eigenstate basis, we have to rotate the couplings appropriately,

$$g_{L,u,d} \rightarrow U_{L,u,d}^\dagger g_{q,u,d} U_{L,u,d} \quad g_{R,u,d} \rightarrow U_{R,u,d}^\dagger g_{-u,-d} U_{R,u,d} \quad (2.16)$$

The rotation introduces non diagonal couplings which lead to tree level contributions to $\Delta F = 2$ processes. Nevertheless, the rotation matrices are hierarchical, with the hierarchy set by the same f_x that controls the SM fermion hierarchies. The off-diagonal KK gluon couplings are of order

$$(g_{L,q})_{ij} \sim g_{s*} f_{q_i} f_{q_j} \quad (g_{R,u})_{ij} \sim g_{s*} f_{-u_i} f_{-u_j} \quad (g_{R,d})_{ij} \sim g_{s*} f_{-d_i} f_{-d_j} \quad (2.17)$$

²The function $\gamma(c)$ is a correction to the approximation used in [6] which can be sizable, e.g. $\gamma(-0.4) \approx 0.52$, $\gamma(0.5) \approx 1.16$, $\gamma(0.7) \approx 1.52$. In numerical calculations we always use the full overlap integral.

The off-diagonal couplings of the quark doublets are suppressed by the ratios of the CKM matrix elements (recall that $f_{q_1} \sim \lambda^3$, $f_{q_2} \sim \lambda^2$). Similarly, the off-diagonal couplings of the singlet quarks are suppressed by hierarchically small entries. This suppression is called the RS-GIM mechanism. It is enough to suppress most of the dangerous $\Delta F = 2$ operators, though not all, as we will see in a moment.

Integrating out the KK gluon and applying appropriate Fierz identities we obtain the effective Hamiltonian:

$$\begin{aligned} \mathcal{H} &= \frac{1}{M_G^2} \left[\frac{1}{6} g_L^{ij} g_L^{kl} (\bar{q}_L^{i\alpha} \gamma_\mu q_{L\alpha}^j) (\bar{q}_L^{k\beta} \gamma^\mu q_{L\beta}^l) \right. \\ &\quad \left. - g_R^{ij} g_L^{kl} \left((\bar{q}_R^{i\alpha} q_{L\alpha}^k) (\bar{q}_L^{l\beta} q_{R\beta}^j) - \frac{1}{3} (\bar{q}_R^{i\alpha} q_{L\beta}^l) (\bar{q}_L^{k\beta} q_{R\alpha}^j) \right) \right] \\ &= C^1(M_G) (\bar{q}_L^{i\alpha} \gamma_\mu q_{L\alpha}^j) (\bar{q}_L^{k\beta} \gamma^\mu q_{L\beta}^l) \\ &\quad + C^4(M_G) (\bar{q}_R^{i\alpha} q_{L\alpha}^k) (\bar{q}_L^{l\beta} q_{R\beta}^j) + C^5(M_G) (\bar{q}_R^{i\alpha} q_{L\beta}^l) (\bar{q}_L^{k\beta} q_{R\alpha}^j) \end{aligned}$$

where α, β are color indices. The Wilson coefficients of these operators will directly correspond to the $C^{1,4,5}$ bounded by the model independent constraints from $\Delta F = 2$ processes by the UTFit collaboration³ in [11], see table 2.

Note, that the most strongly constrained quantity is the imaginary part of C_K^4 for the kaon system. Contributions to ϵ_K coming from C_K^4 are enhanced compared to the ones to C_K^1 with SM like chirality by

$$\sim \frac{3}{4} \left(\frac{m_K}{m_s(\mu_L) + m_d(\mu_L)} \right)^2 \eta_1^{-5} \quad (2.18)$$

where the first factor ≈ 18 is the chiral enhancement of the hadronic matrix element and $\eta_1^{-5} \approx 8$ is the relative RGE running [23].

We are ready to estimate the flavor bounds of the RS model. Using the expressions for the orders of magnitudes for the rotation matrices U we approximately find for the Wilson coefficient at the TeV scale

$$C_{4K}^{\text{RS}} \sim \frac{g_{s*}^2}{M_G^2} f_{q_1} f_{q_2} f_{-d_1} f_{-d_2} \sim \frac{1}{M_G^2} \frac{g_{s*}^2}{Y_*^2} \frac{2m_d m_s}{v^2}. \quad (2.19)$$

Above, m_d, m_s are the down, strange masses at the TeV scale, see table 1. At tree level and in absence of boundary kinetic terms, the bulk coupling g_{s*} is connected the strong coupling at the KK scale by $g_{s*} = g_s(M_G) \log^{1/2}(R'/R) \sim 6$. If arbitrary large Y_* was allowed the bounds from C_{4K} could be eliminated completely. However, if Y_* is too large one loses perturbative control over the theory. One can estimate the upper bound on Y_* using naive dimensional analysis. The proper brane localized Yukawa coupling Y_{5D} is in our normalization $Y_{5D} = Y_* R'$. One-loop corrections to the localized Yukawa coupling would be proportional to $Y_{5D}(Y_{5D}E)^2/(16\pi^2)$, where the energy dependence is inferred from the dimension -1 of Y_{5D} . In order for this to be smaller than the tree-level term we need to impose $(Y_{5D}E)^2/(16\pi^2) \lesssim 1$. We require that this bound is not violated until we reach the

³We are grateful to Luca Silvestrini for discussions about the proper interpretation of the UTFit bounds.

Parameter	Limit on Λ_F (TeV)	Suppression in RS (TeV)
$\text{Re}C_K^1$	$1.0 \cdot 10^3$	$\sim r\sqrt{6}/(V_{td}V_{ts} f_{q_3}^2) = 23 \cdot 10^3$
$\text{Re}C_K^4$	$12 \cdot 10^3$	$\sim r(vY_*)/(\sqrt{2m_d m_s}) = 22 \cdot 10^3$
$\text{Re}C_K^5$	$10 \cdot 10^3$	$\sim r(vY_*)\sqrt{3}/(\sqrt{2m_d m_s}) = 38 \cdot 10^3$
$\text{Im}C_K^1$	$15 \cdot 10^3$	$\sim r\sqrt{6}/(V_{td}V_{ts} f_{q_3}^2) = 23 \cdot 10^3$
$\text{Im}C_K^4$	$160 \cdot 10^3$	$\sim r(vY_*)/(\sqrt{2m_d m_s}) = 22 \cdot 10^3$
$\text{Im}C_K^5$	$140 \cdot 10^3$	$\sim r(vY_*)\sqrt{3}/(\sqrt{2m_d m_s}) = 38 \cdot 10^3$
$ C_D^1 $	$1.2 \cdot 10^3$	$\sim r\sqrt{6}/(V_{ub}V_{cb} f_{q_3}^2) = 25 \cdot 10^3$
$ C_D^4 $	$3.5 \cdot 10^3$	$\sim r(vY_*)/(\sqrt{2m_u m_c}) = 12 \cdot 10^3$
$ C_D^5 $	$1.4 \cdot 10^3$	$\sim r(vY_*)\sqrt{3}/(\sqrt{2m_u m_c}) = 21 \cdot 10^3$
$ C_{B_d}^1 $	$0.21 \cdot 10^3$	$\sim r\sqrt{6}/(V_{tb}V_{td} f_{q_3}^2) = 1.2 \cdot 10^3$
$ C_{B_d}^4 $	$1.7 \cdot 10^3$	$\sim r(vY_*)/(\sqrt{2m_b m_d}) = 3.1 \cdot 10^3$
$ C_{B_d}^5 $	$1.3 \cdot 10^3$	$\sim r(vY_*)\sqrt{3}/(\sqrt{2m_b m_d}) = 5.4 \cdot 10^3$
$ C_{B_s}^1 $	30	$\sim r\sqrt{6}/(V_{tb}V_{ts} f_{q_3}^2) = 270$
$ C_{B_s}^4 $	230	$\sim r(vY_*)/(\sqrt{2m_b m_s}) = 780$
$ C_{B_s}^5 $	150	$\sim r(vY_*)\sqrt{3}/(\sqrt{2m_b m_s}) = 1400$

Table 2: Lower bounds on the NP flavor scale Λ_F for arbitrary NP flavor structure from [11] and the effective suppression scale in RS for KK mass with $M_G = 3$ TeV. Since the Wilson coefficients in [11] are given at the scale Λ_F , we have corrected for the renormalization group scaling from Λ_F to 3 TeV using the expressions in [23] when necessary. We have set $|Y_*| \sim 3$, $f_{q_3} = 0.3$ and $r = M_G/g_{s^*}$.

energies N KK modes, $E = N_{\text{KK}}m_{\text{KK}}$. Using $m_{\text{KK}} \sim 2/R'$ we find that $Y_* \lesssim (2\pi)/N_{\text{KK}}$. For the most conservative bound we set $N_{\text{KK}} = 2$, which imposes $Y_* \lesssim 3$. Thus in our estimates we assume $|Y_*| \lesssim 3$. The suppression scale of the four-fermion operator is set by the lightest KK gluon mass M_G . The RS-GIM mechanism effectively raises the suppression scale by the factor $v/\sqrt{m_d m_s} \sim 10^4$. However, that factor turns out to be an order of magnitude too small for a ~ 3 TeV KK gluon. One can find that in order for the suppression scale to match $1.6 \cdot 10^5$ TeV we need $M_G \sim (22 \pm 6)$ TeV. Our estimate is less optimistic than that encountered in the RS literature so far [10, 24, 25]. The quoted error comes from the uncertainty in the m_d and m_s masses. Including the contributions of the full KK gluon tower may change our result by less than 10%.

In order to understand the actual flavor bound on the RS model in more detail we have generated a sample of points with randomly chosen values of $1/R'$ and brane Yukawa

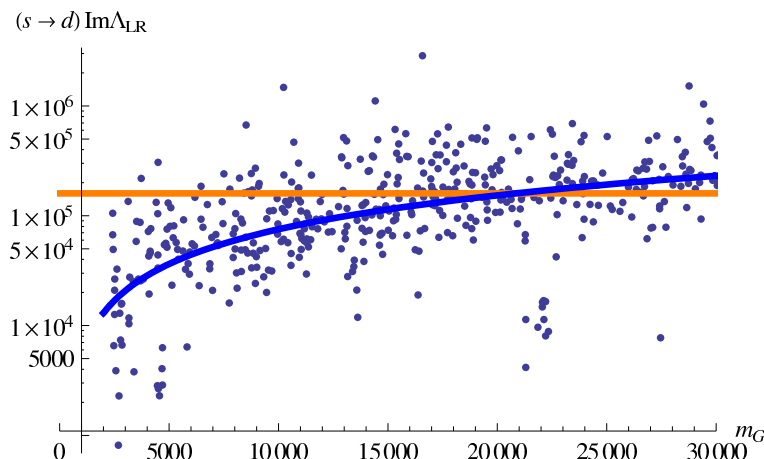


Figure 1: Scan of the effective suppression scale of $\text{Im}C_{4K}$ in the RS model. All the points give the correct low-energy spectrum but most of the points with $m_G < 21$ TeV fail to satisfy the bound of $\Lambda > 1.6 \cdot 10^5$ TeV. The blue line is a linear fit of the M_G dependence. We have taken $c_{q_3} \in [0.4, 0.45]$, $c_{u_3} \in [-0.3, -0.05]$, $c_{d_3} = -0.55$ and $|Y_*| \in [1, 3]$.

couplings of which we selected 500 where the masses, the absolute values of the CKM elements and the Jarlskog invariant approximately matches the SM prediction. We then calculate the exact flavor suppression scale for the C_{4K} operator. The result is presented in figure 1. We can say that, in accordance with our analytical estimates, as long as M_G is below 21 TeV the majority of the generated points violate the flavor bound. This turns the "coincidence problem" of RS [6] into a fine tuning problem: unless there is some additional flavor structure one is likely to violate the flavor bounds.

Note, that there is a model dependence that can strengthen or weaken the above obtained bound: the matching of the bulk gauge coupling to the strong coupling can be changed by adding localized kinetic terms for the gluon

$$\frac{1}{g_s^2(q)} = \frac{\log R'/R}{g_{s*}^2} + \frac{1}{g_{s,UV}^2(q)} + \frac{1}{g_{s,IR}^2(q)} \tag{2.20}$$

A positive brane kinetic term would make the KK gluon more strongly coupled, which would make the flavor bounds more severe. However, the UV brane coupling can be effectively negative at the TeV scale, if one includes the 1-loop running effects [38],⁴

$$\frac{1}{g_{s,UV}^2(q)} = \frac{1}{g_{s,UV}^2(1/R)} - \frac{b_3^{UV}}{8\pi^2} \log(1/qR) \tag{2.21}$$

where b_3^{UV} is the QCD beta functions of the zero modes localized around the UV branes. The running of the IR brane localized kinetic term is cut off at the scale $\sim 1/R'$ therefore it will not involve a large logarithm and we will neglect the IR brane localized terms, and focus only on the UV brane localized kinetic terms. Assuming that the top is localized on

⁴We thank Kaustubh Agashe and Roberto Contino for pointing this out.

the TeV brane, and all other fields on the UV brane for QCD we find $b_3^{UV} = 8$. Thus, the asymptotically free QCD running reduces the magnitude of g_{s*} and thus the coefficient of the operators induced by the KK gluon exchange, while a bare UV brane localized kinetic term would enhance the effect. Our 21 TeV bound presented above corresponds to a choice of boundary kinetic terms where the bare UV couplings exactly cancels the contribution from the running. Another possibility would be to assume no UV boundary kinetic term at the Planck scale. In that case the coupling of the KK gluon is much weaker, changes from $g_{s*} \approx 6$ to $g_{s*} \approx 3$, and the bound is reduced by a factor of ~ 2 from 21 TeV to 10.5 TeV. Yet another possibility is to pick a large bare UV coupling such that the bulk is strongly coupled, $g_{s*} \sim 4\pi$. This would enhance the flavor bound on the KK gluon mass by a factor of ~ 2 from 21 TeV to 42 TeV.

In the remainder of this paper we investigate how the flavor bounds are modified in 5D models where the electroweak breaking sector arises dynamically.

3. 5D models for pseudo-goldstone Higgs

In this section we introduce the 5D framework of GHU and define particular models whose flavor structure we will later study. The basic ingredient of GHU is the presence of a zero mode along the scalar (A_z) direction of the bulk gauge field. This mode is identified with the SM Higgs boson. The idea of GHU has a long history going back to Manton and Hosotani [15]. More recently, GHU has been formulated in 5D warped space-time [26, 16, 17] which, among other things, allows one to obtain a heavy enough top quark. Other important developments are related to electroweak precision observables (EWPO's). The S-parameter is within the experimental bounds if the KK scale of the theory is raised to 2-3 TeV [17]. The ρ -parameter can be protected by custodial symmetry, and the discrete $L \leftrightarrow R$ symmetry protecting the $Zb\bar{b}$ vertex can be implemented [28].

The simplest GHU model with custodial symmetry is based on the $SO(5) \times U(1)_X$ (plus the color group) bulk gauge group which is broken via boundary conditions to the SM group $SU(2)_L \times U(1)_Y$ on the UV brane and to $SO(4) \times U(1)_X$ on the IR brane, where $SO(4) \sim SU(2)_L \times SU(2)_R$. The fact that the $SO(5)/SO(4)$ coset generators are broken on both branes results in four zero modes with the quantum numbers of the SM Higgs doublet. At tree-level, these modes are massless due to 5D gauge invariance, but at one loop they develop a potential.

We denote the dimensionful gauge coupling of $SO(5)$ by $g_* R^{1/2}$. A useful measure of the magnitude of g_* is the number of colors of the dual CFT by

$$N_{\text{CFT}} = \frac{16\pi^2}{g_*^2} \tag{3.1}$$

The 5D description is perturbative as long as $N_{\text{CFT}} \gg 1$. We also allow for a UV brane localized gauge kinetic terms for $SU(2)_L$ (for simplicity, we set the $U(1)_Y$ brane kinetic term to zero) and parametrize it by $1/g_{UV}^2 = r^2 \log(R'/R)/g_*^2$. The $SU(2)_L$ coupling is then related to the 5D parameters by

$$g = \frac{g_*}{\log^{1/2}(R'/R)\sqrt{1+r^2}} \tag{3.2}$$

For a small brane kinetic term, $r \ll 1$, and the Planck-TeV hierarchy, $\log(R'/R) \sim 37$, we need the bulk coupling $g_* \approx 4$ (corresponding to $N_{\text{CFT}} = 10$) in order to reproduce the SM weak coupling. With the brane kinetic term we have to make the bulk more strongly coupled, in particular for $r^2 = 4$ we go down to $N_{\text{CFT}} = 2$.

The four components h^a of the Higgs doublet are embedded into the gauge field A_5 as

$$A_z(z) = \sqrt{\frac{2}{R}} \frac{z}{R'} T_C^a h^a(x) \tag{3.3}$$

Here, the T_C^a 's are the generators of the $\text{SO}(5)/\text{SO}(4)$ coset. The normalization is chosen such that h^a 's have the canonical kinetic terms in the 4D effective theory. We fix the Higgs vev along the h_4 direction, and we denote $\langle h_4 \rangle = \tilde{v}$.

The 5D model has two important scales that play a vital role in the dynamics. One is R' which sets the KK scale, and the masses of the lightest KK modes of electroweak gauge bosons are roughly $\sim 2.4R'$. Another important quantity is the global symmetry breaking scale f_π (or the "Higgs decay constant") defined by

$$f_\pi = \frac{2}{g_* R'} = \frac{2}{g \sqrt{\log R'/R} \sqrt{1+r^2} R'}. \tag{3.4}$$

Because the logarithm is large, $f_\pi < R'$ by at least a factor of ~ 2 . The W-mass is connected to the scale f_π and the Higgs vev via

$$M_W^2 = \frac{g^2 f_\pi^2}{4} \sin^2(\tilde{v}/f_\pi) \tag{3.5}$$

For $\tilde{v} \ll f_\pi$, one recovers the SM formula $M_W^2 \approx g^2 \tilde{v}^2/4$.

Fixing the gauge group still leaves several options for realizing the fermionic sector of the theory, depending on how the SM quarks are embedded into $\text{SO}(5)$ representations. In this paper we consider three distinct realizations that have previously appeared in the literature. Before we move to the detailed description of the model we point a few general model-building rules that we need follow.

- *The SM quarks are identified with the zero modes of 5D quarks.* The presence of zero modes depends on the boundary conditions. There are two possibilities: either we choose the right-handed chirality of the 5D fermion to vanish on a boundary (which is denoted as [+]), or the left-handed chirality vanishes (denoted as [-]). Left-handed zero modes - appropriate for SM doublet quarks - arise when the right-handed chirality vanishes on both the UV and the IR brane, the choice denoted as [++]. The [--] boundary conditions lead to right-handed zero modes appropriate for the $\text{SU}(2)_L$ singlet SM quarks.
- *Each of the SM quarks should be embedded in a separate $\text{SO}(5)$ multiplet.* In principle, $\text{SO}(5)$ multiplets contain fields with the quantum numbers of both doublet and singlet quarks. However, multiplets that yield both doublet left-handed zero modes and singlet right-handed zero modes are problematic for the following reason. For the first two generations, if the left-handed zero mode is localized close to the UV brane,

then the right handed zero mode is localized at the IR brane, which leads to problems with precision measurements. For the third generation, the reason is more subtle and has to do with the radiative generation of the Higgs potential; we will comment on that later. Thus, we need at least three $SO(5)$ multiplets for each generation. As a consequence, the field content of GHU models is necessarily larger than that of RS.

- Apart from the zero mode, the remaining fields in the multiplets should yield only heavy KK modes. One should take care that the boundary conditions do not lead to ultra-light KK modes. This may happen for the quarks living in the same multiplet with UV localized zero modes, depending on the boundary conditions of the remaining fields. The rule of thumb is that $SO(5)$ partners of $[++]$ fields should be assigned $[-+]$ boundary conditions, while the partners of $[--]$ fields should have $[+-]$.
- This zeroth-order picture is modified by the mass terms on the IR brane that mix different 5D multiplets. These mass terms are necessary to arrive at acceptable phenomenology. First of all, since the Higgs field is a component of A_z , it only couples 5D quarks from the same bulk multiplet. In order to obtain non-zero quark masses, at least some of the zero modes should have non-vanishing components in more than one multiplet. The boundary mass terms play a similar role as the Yukawa couplings in RS but, as we discuss later, there are some important differences.
- At the end of the day, the boundary conditions to $SO(5)$ multiplets should be such that *SO(5) is broken on both the UV and the IR branes*. The Wilson-line breaking is a non-local effect that is operating only when the gauge symmetry is broken on both endpoints of the fifth dimension. If either the UV or the IR boundary conditions are $SO(5)$ symmetric, the Wilson line can be rotated away, and the SM quarks do not acquire masses.

We move to discussing three specific realizations that satisfy the above requirements.

3.1 Spinorial

The spinor representation $\mathbf{4}$ is the smallest $SO(5)$ representation. Although models with the third generation embedded in the spinorial representation have severe problems with satisfying the precision constraints on the Zbb vertex, we do include it in our study. The reason is that this model is the simplest (it has the minimal number of bulk fields), and its flavor structure is most transparent. Furthermore, almost identical flavor structure appears in the fully realistic models.

We consider 3 bulk $SO(5)$ spinors for a single generation of quarks, Ψ_q, Ψ_u, Ψ_d (recall that we omit the generation index; all fermionic fields should be read as three-vectors in the generation space). Under the $SU(2)_L \times SU(2)_R$ subgroup it splits as $4 \rightarrow (2, 1) + (1, 2)$, so that an $SO(5)$ spinor contains both $SU(2)_L$ doublets and singlets. Roughly, Ψ_q will provide the zero mode for the left-handed quark doublets, while Ψ_u, Ψ_d for the right handed up and down-type quarks. To obtain the SM zero mode spectrum we impose the following

boundary conditions

$$\Psi_q = \begin{pmatrix} q_q[+, +] \\ u_q^c[-, +] \\ d_q^c[-, +] \end{pmatrix} \quad \Psi_u = \begin{pmatrix} q_u[+, -] \\ u_u^c[-, -] \\ d_u^c[+, -] \end{pmatrix} \quad \Psi_d = \begin{pmatrix} q_d[+, -] \\ u_d^c[+, -] \\ d_d^c[-, -] \end{pmatrix} \quad (3.6)$$

with the notation that the first component is a complete $SU(2)_L$ doublet, while the lower two components are the two components of an $SU(2)_R$ doublet, $q^c = (u^c, d^c)$. Our model is similar to the one in ref. [17], even though we assign different IR boundary conditions for the $SU(2)_R$ doublets.⁵

We denote the left-handed chirality modes of a Dirac field by χ , while the right-handed chiralities by ψ . For example χ_{q_q} stands for the left-handed chirality $SU(2)_L$ doublet contained in Ψ_q . The above set of parity assignments ensures the zero modes with SM quantum numbers in $\chi_{q_q}, \psi_{u_u^c}$ and $\psi_{d_d^c}$. However, at this point there would be no Yukawa couplings at all, since the zero modes live in completely different bulk multiplets. To obtain non-zero Yukawa couplings, at least some of the zero modes should have non-vanishing components in more than one multiplet. This can be achieved via the following IR localized mass terms:

$$\mathcal{L}_{\text{IR}} = - \left(\frac{R}{R'} \right)^4 \left[\tilde{m}_u \chi_{q_q} \psi_{q_u} + \tilde{m}_d \chi_{q_q} \psi_{q_d} + \tilde{M}_u (\chi_{u_u^c} \psi_{u_u^c} + \chi_{d_u^c} \psi_{d_u^c}) + \tilde{M}_d (\chi_{u_d^c} \psi_{u_d^c} + \chi_{d_d^c} \psi_{d_d^c}) \right] \quad (3.7)$$

Here $\tilde{m}_{u,d}, \tilde{M}_{u,d}$ are dimensionless 3 by 3 matrices, which will play the similar role as brane-localized Yukawa couplings in the original RS. All flavor mixing effects in this model originate from the IR localized mass terms. The effect of \tilde{m}_u is to rotate the doublet zero mode partly into the Ψ_u field, while \tilde{m}_d rotates it partly into Ψ_d . At the same time \tilde{M}_u will rotate the singlet up-type zero mode partly into Ψ_q , and similarly \tilde{M}_d will rotate the down-type zero mode into Ψ_d . The boundary mass terms respect the $SU(2)_L \times SU(2)_R$ of the IR brane, but they break $SO(5)$. In the limit $\tilde{m}_u = \tilde{M}_u$ and $\tilde{m}_d = \tilde{M}_d$ $SO(5)$ invariance is restored in the IR boundary conditions, and the zero mode quarks become massless.

In the presence of the boundary terms the IR brane boundary conditions will be modified as

$$\begin{aligned} \psi_{q_q} &= -\tilde{m}_u \psi_{q_u} - \tilde{m}_d \psi_{q_d} \\ \chi_{q_u} &= \tilde{m}_u^\dagger \chi_{q_q} \\ \chi_{q_d} &= \tilde{m}_d^\dagger \chi_{q_q} \end{aligned} \quad (3.8)$$

$$\begin{aligned} \psi_{Q_q} &= -\tilde{M}_u \psi_{Q_u} - \tilde{M}_d \psi_{Q_d} \\ \chi_{Q_u} &= \tilde{M}_u^\dagger \chi_{Q_q} \\ \chi_{Q_d} &= \tilde{M}_d^\dagger \chi_{Q_q} \end{aligned} \quad (3.9)$$

3.2 Fundamental + Adjoint

As we explain later, the model with fermions in the spinor representation turns out to have incurable problems: the Higgs mass tends to be too light, and there is a large irreducible

⁵In particular, Ψ_q and Ψ_u alone would be equivalent after interchanging $\tilde{M}_u \rightarrow 1/\tilde{M}_u$. With Ψ_d included, the two models are not equivalent.

correction to the $Zb\bar{b}$ vertex. The situation is improved in models where the doublet quarks are embedded in the fundamental representation of $SO(5)$. The original motivation for considering the fundamental representation was the realization [18] that it is possible to greatly reduce the corrections to the $Zb\bar{b}$ vertex by using an embedding of the SM fermions into the custodially symmetric $SU(2)_L \times SU(2)_R$ model under which the b_L is symmetric under $SU(2)_L \leftrightarrow SU(2)_R$. The simplest implementation of this Z_2 symmetry is when the left handed quarks are in a bifundamental under $SU(2)_L \times SU(2)_R$, while t_R is a singlet.

In the context of the $SO(5)$ MCH model the minimal model is obtained via introducing two fundamental (**5**) and one adjoint (**10**) representation. This is the model for which the constraints from electroweak symmetry breaking and electroweak precision constraints have been investigated in detail in [27]. Under $U(1)_X$ all bulk fermion carry charge $-2/3$. Under the $SO(4)$ subgroup $5 \rightarrow (2, 2) + (1, 1)$, while $10 \rightarrow (3, 1) + (1, 3) + (2, 2)$. In order to protect the $Zb\bar{b}$ vertex we want q_L to be part of $(2, 2)$, t_R in $(1, 1)$ and b_R in $(1, 3)$. Thus the fermion content of this model will be 2×3 5D quarks in the fundamental representation and 1×3 quarks in the adjoint representation of $SO(5)$. The fives (denoted Ψ_q, Ψ_u) each host three up quarks u, \tilde{u}, u^c , one down quark d and one exotic charge $5/3$ quark X . $q = (u, d)$ is hypercharge $1/6$ $SU(2)_L$ doublet, while $\tilde{q} = (X, \tilde{u})$ is hypercharge $-7/6$ $SU(2)_L$ doublet. (q, \tilde{q}) is a bidoublet under $SU(2)_L \times SU(2)_R$. The tens (denoted Q_d) hosts four up quarks $u^d, \tilde{u}^d, u^l, u^r$, three down quarks d^d, d^l, d^r , and three exotics X^d, X^l, X^r . These quarks are collected into an $SU(2)_L$ triplet $l = (X^l, u^l, d^l)$, an $SU(2)_R$ triplet $r = (X^r, u^r, d^r)$ and a bidoublet q^d, \tilde{q}^d , where $q^d = (u^d, d^d)$ is hypercharge $1/6$ $SU(2)_L$ doublet, while $\tilde{q}^d = (X^d, \tilde{u}^d)$ is hypercharge $-7/6$ $SU(2)_L$ doublet. Below we write the appropriate boundary conditions for the left-handed fields of every component:

$$\Psi_q = \begin{pmatrix} q_q[+, +] & \tilde{q}_q[-, +] \\ u_q^c[-, +] & \end{pmatrix} \quad \Psi_u = \begin{pmatrix} q_u[+, -] & \tilde{q}_u[+, -] \\ u_u^c[-, -] & \end{pmatrix} \quad (3.10)$$

$$\Psi_d = \begin{pmatrix} l[+, -] & r = \begin{pmatrix} X_r[+, -] \\ u_r[+, -] \\ d_r[-, -] \end{pmatrix} \\ q_d[+, -] & \tilde{q}_d[+, -] \end{pmatrix} \quad (3.11)$$

With the above parity assignments we can also add IR boundary masses for the fermions, which are necessary to generate the effective Yukawa couplings. These are given by

$$\mathcal{L}_{\text{IR}} = - \left(\frac{R}{R'} \right)^4 \left[\tilde{m}_u (\chi_{q_q} \psi_{q_u} + \chi_{\tilde{q}_q} \psi_{\tilde{q}_u}) + \tilde{M}_u \chi_{u_q^c} \psi_{u_q^c} + \tilde{m}_d (\chi_{q_d} \psi_{q_d} + \chi_{\tilde{q}_d} \psi_{\tilde{q}_d}) \right] + \text{h.c.} \quad (3.12)$$

Note that the symmetries allow for one less boundary mass matrix than in the spinorial model. The flavor structure of this model turns out to be identical to that of the spinorial model with $\tilde{M}_d = 0$.

3.3 Four-fundamental

Another possible implementation of the symmetry protecting the $Zb\bar{b}$ vertex is to use four copies of bulk fundamentals for every generation: two for the up-type quarks and two for

the down-type quarks [28]. In the up sector one of the fundamental provides a left handed doublet zero mode (in the bifundamental of the custodial symmetry), and one right handed up-type singlet zero mode. In order to obtain the correct hypercharges for the SM quarks, the $U(1)_X$ charge of these up-type fundamentals has to be $-2/3$. To realize the down sector we need two additional fundamentals, with X charge $1/3$, one providing another doublet zero mode, and the other the down right zero mode. In order to remove the additional doublet, we must assume that on the Planck brane there is an additional right handed doublet, which will marry one combination of the two doublet zero modes.

The boundary conditions are given by⁶

$$\Psi_{q_u} = \begin{pmatrix} q_{q_u}[\pm, +] & \tilde{q}_{q_u}[-, +] \\ u_q[-, +] & \end{pmatrix} \quad \Psi_u = \begin{pmatrix} q_u[+, -] & \tilde{q}_u[+, -] \\ u_u[-, -] & \end{pmatrix} \quad (3.13)$$

$$\Psi_{q_d} = \begin{pmatrix} \tilde{q}_{q_d}[-, +] & q_{q_d}[\pm, +] \\ d_q[-, +] & \end{pmatrix} \quad \Psi_d = \begin{pmatrix} \tilde{q}_d[+, -] & q_d[+, -] \\ d_d[-, -] & \end{pmatrix} \quad (3.14)$$

Here $[\pm]$ stands for mixed boundary conditions for the electroweak doublets q_{q_u} and q_{q_d} on the UV brane:

$$\theta \chi_{q_{q_u}} - \chi_{q_{q_d}} = 0 \quad \psi_{q_{q_u}} + \theta^\dagger \psi_{q_{q_d}} = 0 \quad (3.15)$$

where θ is a 3×3 matrix that describes which combinations of the fields $\chi_{q_{q_u}}$ and $\chi_{q_{q_d}}$ are removed on the UV brane.

Then the left handed zero modes from $[[+]]$ and the right handed zero modes from $[-]$ fields are all elementary. We again add the IR boundary mass terms

$$- \left(\frac{R}{R'} \right)^4 \left[\tilde{m}_u (\chi_{q_{q_u}} \psi_{q_u} + \chi_{\tilde{q}_{q_u}} \psi_{\tilde{q}_u}) + \tilde{M}_u \chi_{u_q} \psi_{u_u} \right. \\ \left. + \tilde{m}_d (\chi_{q_{q_d}} \psi_{q_d} + \chi_{\tilde{q}_{q_d}} \psi_{\tilde{q}_d}) + \tilde{M}_d \chi_{d_q} \psi_{d_d} \right] + \text{h.c.} \quad (3.16)$$

We have four IR matrices, just like in the spinorial model. In this model, however, there is an additional source of flavor violation - the matrix θ that sets the UV boundary conditions.

4. Higgs potential

In this section we review the computation of the one-loop Higgs potential in the GHU models considered in this paper. Determining the shape of the potential will allow us to pinpoint the regions of parameter space that lead to a correct electroweak breaking vacuum. We will later use this input in our studies of the parameter space allowed by flavor constraints. This section is more technical and slightly outside the main line of the paper, yet we include it to keep the paper self-contained. Those readers whose primary interest is in flavor physics are cordially invited to jump straight to the next section.

⁶Again, our IR boundary conditions are different than those of ref. [28] but the physical content of the model is similar. In particular, the up-quark sector alone would be equivalent after replacing \tilde{M}_u with $1/\tilde{M}_u$.

A radiative Higgs potential is generated at one-loop level because the tree-level KK mode masses depend on the vev \tilde{v} of the Wilson line. The simplest way to calculate is to use the so-called spectral function $\rho(p^2) = \det(-p^2 + m_n^2(\tilde{v}))$, that is a function of 4D momenta whose zeros encode the whole KK spectrum in the presence of the electroweak breaking. With a spectral function at hand, we can compute the Higgs potential from the Coleman-Weinberg formula,

$$V(\tilde{v}) = \frac{N}{(4\pi)^2} \int_0^\infty dp p^3 \log(\rho[-p^2]) \quad (4.1)$$

where $N = -4N_c$ for quark fields and $N = +3$ for gauge bosons. The spectral function can be computed by solving the equations of motion and the boundary conditions in the presence of the Higgs vev.

The leading contribution to the Higgs potential comes from the top quark sector and the gauge sector. Thus, we can restrict to computing the spectral functions of the top quark KK tower (ρ_t), the W boson tower (ρ_W) and the Z boson tower (ρ_Z). For the sake of this computation we ignore the mixing of the top quark with the first two generations. An explicit expression for the potential in terms of these spectral functions is (with $t = p^2$):

$$V(\tilde{v}) = \frac{3}{32\pi^2} \int_0^\infty dt t [-4 \log \rho_t(-t) + 2 \log \rho_W(-t) + \log \rho_Z(-t)]. \quad (4.2)$$

The gauge sector is common to all three models. The spectral functions for the SO(5) GHU model were already given in detail in [29, 31]. For completeness we will summarize the results below. Of the various SO(5) gauge bosons only the masses of the tower corresponding to the W,Z bosons depends on the Higgs VEV. The spectral function has the form

$$\rho_{W,Z}(-p^2) = 1 + f_{W,Z}(-p^2) \sin^2(\tilde{v}/f_\pi), \quad (4.3)$$

where the form factors $f^{W,Z}$ do not depend on \tilde{v} . The equations of motion in AdS are solved in term of the Bessel functions, and the form factors turn out to be complicated combinations thereof. There is however a way to organize them in a more convenient form by using the generalized warped-space trigonometric functions introduced in ref. [34]. We define $C(z)$ and $S(z)$ to be the two independent solution of the equations of motion that satisfy the UV boundary conditions $C(R) = 1$, $C'(R) = 0$, $S(R) = 0$, $S'(R) = m$. Explicitly they read,

$$\begin{aligned} C(z) &= \frac{\pi m z}{2} [Y_0(mR) J_1(mz) - J_0(mR) Y_1(mz)] \\ S(z) &= \frac{\pi m z}{2} [-Y_1(mR) J_1(mz) + J_1(mR) Y_1(mz)] \end{aligned} \quad (4.4)$$

Using this, the form factors can be abbreviated to

$$\begin{aligned} f_W(m^2) &= \frac{m}{2} \left(\frac{R'}{R} \right) \frac{1}{[C'(R') - r^2 m R \log(R'/R) S'(R')] S(R')} \\ f_Z(m^2) &= \frac{m}{2} \left(\frac{R'}{R} \right) \frac{1 + \frac{\tan^2 \theta_W}{1+r^2} \left(1 - r^2 m R \log(R'/R) \frac{S'(R')}{C'(R')} \right)}{[C'(R') - r^2 m R \log(R'/R) S'(R')] S(R')} \end{aligned} \quad (4.5)$$

where $\tan \theta_W = g'/g$. In the evaluation of the Higgs potential we use the warped trigonometric functions at $m^2 = -p^2$, e.g.

$$S(R') = ipR' [-I_1(pR) K_1(pR') + K_1(pR) I_1(pR')] . \quad (4.6)$$

We move to the top sector. Again, the equations of motions can be solved in terms of the Bessel functions, but there is also a dependence on the bulk mass parameters c . As for the gauge bosons, it is convenient to introduce the AdS warped trigonometric functions (here already evaluated for $m^2 = -p^2$)

$$\begin{aligned} C_c &= pR \left(\frac{R}{R'}\right)^{-c-1/2} [K_{c-1/2}(pR) I_{c+1/2}(pR') + I_{c-1/2}(pR) K_{c+1/2}(pR')] \\ S_c &= pR \left(\frac{R}{R'}\right)^{-c-1/2} [-I_{c+1/2}(pR) K_{c+1/2}(pR') + K_{c+1/2}(pR) I_{c+1/2}(pR')] \end{aligned} \quad (4.7)$$

The form of the spectral function depends on the fermion representations, and we have to treat each of the three models separately. In the **spinorial** model, the spectral function can then be parameterized in terms of the form factors $f_{2,4}$

$$\rho_t(-p^2) = 1 + f_2^t(-p^2) \sin^2(\tilde{v}/2f_\pi) + f_4^t(-p^2) \sin^4(\tilde{v}/2f_\pi) \quad (4.8)$$

Note that the spectral depends on $\sin^2(\tilde{v}/2f_\pi)$ rather than $\sin^2(\tilde{v}/f_\pi)$, which is a peculiarity of the spinorial representation. The form factors can be written as

$$\begin{aligned} f_2^t &= \frac{F_2^t}{F_0^t} \\ f_4^t &= \frac{F_4^t}{F_0^t} \\ F_0(p^2) &= \left[S_{-c_q} C_{c_u} + |\hat{m}_u^2| S_{-c_u} C_{c_q} + |\hat{m}_d^2| S_{-c_d} \frac{C_{c_u} C_{c_q}}{C_{c_d}} \right] \times \\ &\quad \times \left[S_{c_u} C_{-c_q} + |\hat{M}_u^2| S_{c_q} C_{-c_u} - |\hat{M}_d^2| S_{-c_d} \frac{S_{c_u} S_{c_q}}{C_{c_d}} \right] \\ F_2(p^2) &= -|\hat{m}_u - \hat{M}_u|^2 + (|\hat{m}_u|^2 - |\hat{M}_u|^2)(S_{c_q} S_{-c_q} - S_{c_u} S_{-c_u}) \\ &\quad - (|\hat{m}_d|^2 - |\hat{M}_d|^2) S_{c_u} S_{-c_d} \frac{C_{c_u}}{C_{c_d}} + (|\hat{m}_d|^2 |\hat{M}_u|^2 - |\hat{m}_u|^2 |\hat{M}_d|^2) S_{c_q} S_{-c_d} \frac{C_{c_q}}{C_{c_d}} \\ F_4(p^2) &= |\hat{m}_u - \hat{M}_u|^2 \end{aligned} \quad (4.9)$$

The hatted boundary masses are again defined as

$$\begin{aligned} \hat{m}_u &= (R'/R)^{c_u - c_q} \tilde{m}_u & \hat{M}_u &= (R'/R)^{c_u - c_q} \tilde{M}_u \\ \hat{m}_d &= (R'/R)^{c_d - c_q} \tilde{m}_d & \hat{M}_d &= (R'/R)^{c_d - c_q} \tilde{M}_d \end{aligned} \quad (4.11)$$

In the model with **fundamentals + adjoint** the spectral functions read

$$\rho_t(m^2) = 1 + f_2^t(m^2) \sin^2(\tilde{v}/f) + f_4^t \sin^4(\tilde{v}/f). \quad (4.12)$$

Note that for the fundamental and adjoint representations the spectrals are expressed in terms of $\sin^2(\tilde{v}/f)$, rather than $\sin^2(v/2f)$ as for the spinors. We write

$$\begin{aligned} f_2^t &= \frac{F_2^t(m^2)}{F_0^t(m^2)} \\ f_4^t &= \frac{F_4^t(m^2)}{F_0^t(m^2)} \end{aligned} \quad (4.13)$$

$$\begin{aligned} F_0^t &= \left[S_{-q}C_u + |\hat{m}_u|^2 C_q S_{-u} + \frac{S_{-d}}{C_d} |\hat{m}_d|^2 C_q C_u \right] \left[C_{-q}S_u + |\hat{M}_u|^2 S_q C_{-u} \right] \\ F_2^t &= \frac{1}{2} \left(-|\hat{m}_u - \hat{M}_u|^2 + (|\hat{m}_u|^2 - |\hat{M}_u|^2)(2S_q S_{-q} - S_u S_{-u}) \right. \\ &\quad \left. - \frac{S_{-d}}{C_d} |\hat{m}_d|^2 (S_u C_u + 2|\hat{M}_u|^2 S_q C_q) \right. \\ &\quad \left. + \frac{|\hat{m}_u|^2 (|\hat{m}_u|^2 - |\hat{M}_u|^2) S_q S_{-u} + \frac{S_{-d}}{C_d} |\hat{m}_u|^2 |\hat{m}_d|^2 S_q C_u}{C_{-q}C_u - |\hat{m}_u|^2 S_q S_{-u} - \frac{S_{-d}}{C_d} |\hat{m}_d|^2 S_q C_u} \right) \\ F_4^t &= \frac{1}{2} |\hat{m}_u - \hat{M}_u|^2 \end{aligned} \quad (4.14)$$

Finally, in the model with **four fundamental** multiplets

$$\begin{aligned} \rho_t(m^2) &= 1 + f_2^t(m^2) \sin^2(\tilde{v}/f) + f_4^t \sin^4(\tilde{v}/f) \\ f_2^t &= \frac{F_2^t(m^2)}{F_0^t(m^2)} \\ f_4^t &= \frac{F_4^t(m^2)}{F_0^t(m^2)} \\ F_0^t(m^2) &= F_0^0 + |\theta|^2 F_0^\theta \\ F_2^t(m^2) &= F_2^0 + |\theta|^2 F_2^\theta \end{aligned} \quad (4.15)$$

We find

$$\begin{aligned} F_0^0 &= [S_{-q_u}C_u + |\hat{m}_u^2|C_{q_u}S_{-u}] [C_{-q_u}S_u + |\hat{M}_u^2|S_{q_u}C_{-u}] \\ F_0^\theta &= [C_{-q_u}S_u + |\hat{M}_u^2|S_{q_u}C_{-u}] [C_{-q_u}C_u - |\hat{m}_u^2|S_{q_u}S_{-u}] \\ &\quad \times [S_{-q_d}C_d + |\hat{m}_d^2|C_{q_d}S_{-d}] / [C_{-q_d}C_d - |\hat{m}_d^2|S_{q_d}S_{-d}] \\ F_2^0 &= \frac{1}{2} \left(-|\hat{m}_u - \hat{M}_u|^2 + (|\hat{m}_u|^2 - |\hat{M}_u|^2)(2S_{q_u}S_{-q_u} - S_u S_{-u}) \right) \\ &\quad + \frac{1}{2} |\hat{m}_u|^2 (|\hat{m}_u|^2 - |\hat{M}_u|^2) S_{q_u} S_{-u} / [C_{-q_u}C_u - |\hat{m}_u^2|S_{q_u}S_{-u}] \\ F_2^\theta &= (|\hat{m}_u|^2 - |\hat{M}_u|^2) C_{-q_u} S_{q_u} \\ &\quad \times [S_{-q_d}C_d + |\hat{m}_d^2|C_{q_d}S_{-d}] / [C_{-q_d}C_d - |\hat{m}_d^2|S_{q_d}S_{-d}] \\ F_4^t &= \frac{1}{2} |\hat{m}_u - \hat{M}_u|^2 \end{aligned} \quad (4.17)$$

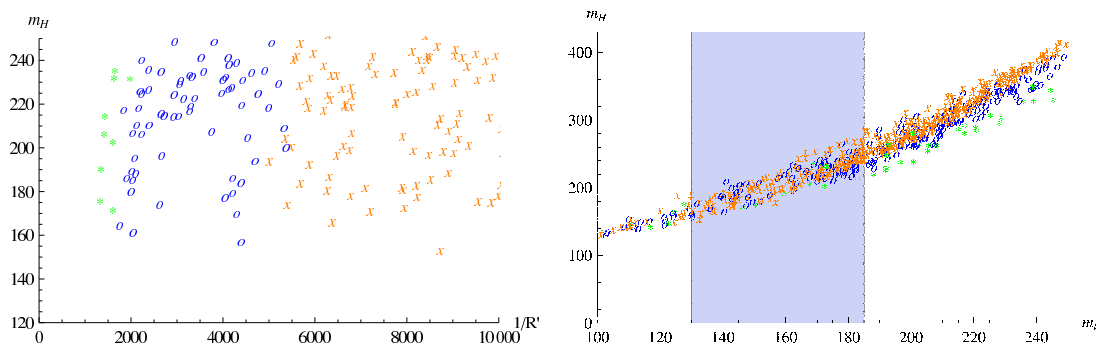


Figure 2: Left panel: the dependence of the Higgs mass on $1/R'$ for a variety of input parameters that give successful electroweak symmetry breaking for the model with bulk fundamentals and adjoints with points selected to give the top mass in the physical range. Right panel: the correlation between the top and the Higgs masses for the same case. Orange (x), blue (o), and green (*) points correspond to $\epsilon < 0.1$, $0.1 < \epsilon < 0.3$, and $0.3 < \epsilon$.

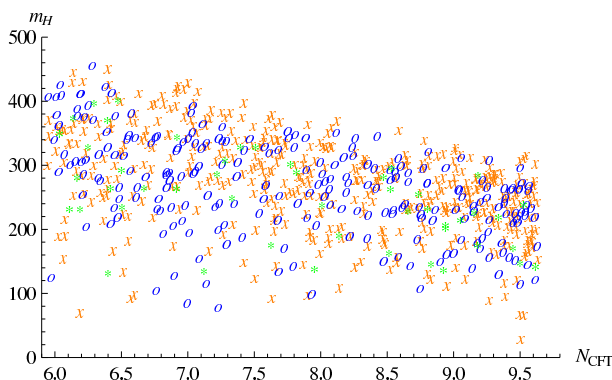


Figure 3: The dependence of the Higgs mass on the effective number of colors N_{CFT} (corresponding strength of the bulk coupling) for the case of bulk fundamental plus adjoint fermions. Orange (x), blue (o), and green (*) points correspond to $\epsilon < 0.1$, $0.1 < \epsilon < 0.3$, and $0.3 < \epsilon$.

These expressions can be used for the calculation of the Higgs potential in each model. One practical option is to numerically calculate the integral in (4.2). However, it is useful to gain some more insight into the shape of the potential. This can be done by expanding the potential in powers of $x \equiv \sin^2(\tilde{v}/f_\pi)$ (for the top contribution in the spinorial representation we expand in $x = \sin^2(\tilde{v}/2f_\pi)$). One complication in this expansion is that the integral in (4.2) is logarithmically IR divergent in the limit $x \rightarrow 0$. This is due to the fact that some of the masses vanish in the limit $\tilde{v} \rightarrow 0$, and the usual CW formula contains terms of the form $m^4 \log m^2$.

The top contribution to the Higgs potential has the form

$$V_t(x) = -\frac{3}{8\pi^2} \int_0^\infty dt t \log[1 + f_2^t(-t)x + f_4^t(-t)x^2] \tag{4.18}$$

For small x this can be expanded as

$$V_t(x) = a_1^t x + a_2^t x^2 + n_2^t x^2 \log \frac{2c_t x}{\Lambda^2} + \mathcal{O}(x^3) \quad (4.19)$$

We defined the coefficient c_t that captures the IR behavior of the form factors, namely $f_1^t(-t) \approx -f_2^t(-t) \approx c_t/t$ for $t \rightarrow 0$, and it is related to the SM top quark mass by $m_t^2 \approx c_t x$. This coefficient can be read off from the top spectral function by replacing $C_c \rightarrow 1$, $S_c \rightarrow itR'(R'/R)^{-2c} f_c^{-2}$. We can then show that the general expression for the coefficients in the expansion should be

$$\begin{aligned} a_1^t &= -\frac{3}{8\pi^2} \int_0^\infty dt t f_2^t \\ a_2^t &= -\frac{3}{16\pi^2} \left[\int_0^\infty dt t \left(2f_4^t - (f_2^t)^2 + \frac{c^2}{\Lambda^4 \sinh^2(t/\Lambda^2)} \right) - \frac{3}{2} c_t^2 \right] \\ n_2^t &= -\frac{3}{16\pi^2} c_t^2 \end{aligned} \quad (4.20)$$

The scale Λ is an IR regulator and it may take an arbitrary value. The expansion parameters depend on Λ in such a way that the dependence on the IR regulator cancels out at order x^2 .

In the cases when the top contribution, dominates the minimum of the potential is given by the approximate formula:

$$x_m = -\frac{a_1^t}{2a_2^t + n_2^t + 2n_2^t \log(2cx_m/\Lambda^2)} \quad (4.21)$$

In practice, however, the acceptable minimum with $x_m \ll 1$ occurs only for fine-tuned values of the parameters such that a_1^t is much smaller than its natural value $\sim m_t^2(R')^2/4\pi^2$. In that case, the W and Z boson contributions can significantly shift the minimum, and the acceptable EW breaking vacuum occurs for different c-parameters than without the gauge contributions. The gauge contributions are calculated from

$$\begin{aligned} V_W(x) &= \frac{3}{16\pi^2} \int_0^\infty dt t \log[1 + f_2^W(-t)x] \\ V_Z(x) &= \frac{3}{32\pi^2} \int_0^\infty dt t \log[1 + f_2^Z(-t)x] \end{aligned} \quad (4.22)$$

and the expansion can be done analogously as for the top, by setting $f_4^{W,Z} = 0$. In our numerical studies we take the gauge contributions into account.

We turn to discussing the main features of the Higgs potential generated by the top, bottom and gauge loops. We have scanned the parameter space to find self-consistent combinations leading to realistic EWSB. In the scan we first choose the brane kinetic term r , the bulk masses c_{q_3} and c_{d_3} , the size of the brane masses \tilde{m}_u , \tilde{m}_d , \tilde{M}_u , \tilde{M}_d , the KK scale $1/R'$ and keep the hierarchy fixed at $R/R' = 10^{-16}$. We then determine v/f_π such that the m_W mass is reproduced and finally choose c_{u_3} such that the minimum of the Coleman-Weinberg potential is really at this value of v/f_π .

For the case of fundamental plus adjoint bulk fermions we do find plenty of realistic values for the Higgs and the top masses, in agreement with the results of [31]. This is illustrated in figure 2. Note however, that in order to fit the W-mass successfully with a sufficiently small ϵ one is usually forced to introduce UV localized kinetic terms. The dependence of the Higgs mass on the UV localized kinetic terms is illustrated in figure 3. For our analysis of the flavor scales we are only selecting from the points that give a satisfactory top with low values of ϵ .

However, there is a fine tuning reminiscent of the little hierarchy problem showing up: if one fixes the mixing parameters $\tilde{m}_{u,d}, \tilde{M}_{u,d}$ and the bulk masses $c_{q,d}$, then for given radii R, R' and for generic choices of the other parameters we have only a very narrow region in the parameter c_u that produces an ϵ which is phenomenologically acceptable (for example $0.1 < \epsilon < 0.45$). This suggests that in the interesting region with proper electroweak symmetry breaking there will be a strong sensitivity of ϵ to the input parameters. This was first pointed out in [30] and we find it to be a general property of all representations studied in this paper. To illustrate this we show two examples of the dependence of ϵ on c_u in figure 4. The first one is a randomly chosen point where one has to adjust c_u to a high precision in order to find proper EWSB, while the second one corresponds to the best case scenario that we could find after searching for regions where the tuning is milder. We can see that the derivative of ϵ is very large in the first point, and even in the second point it is still quite sizeable.

In order to quantify the tuning in these models we have calculated the local sensitivity to the input parameters in the ranges given in section 5.3, for the regions with acceptable electroweak symmetry breaking only. We then define the local fine tuning as [32, 35]

$$t^{-1} = \max \left| \frac{\partial \log \epsilon}{\partial \log a_i} \right| \tag{4.23}$$

In this scan we have only included points where EWSB happens with a sufficiently small S parameter and excluded all other points. The parameters a_i were taken to be $c_{q3}, c_{u3}, \tilde{m}_u, \tilde{M}_u$. We then estimate the average tuning for given ϵ by fitting the average of t with a quadratic polynomial in ϵ . The best fit is approximately

$$t \sim \frac{1}{4} \epsilon^2 \tag{4.24}$$

which qualitatively agrees with the ϵ^2 estimate of [17] but is numerically somewhat stronger. This implies that the average local tuning is about half a percent for $\epsilon = 0.1$, while about 5% for $\epsilon = 0.4$. To verify those estimates we extended the parameter space to a large grid in the parameters $c_{q3}, c_{u3}, \tilde{m}_u, \tilde{M}_u$ and checked that the above fit for $t = t(\epsilon)$ remains a conservative lower bound over all parameter space. These averages are shown as crosses in figure 5 where each cross contains the average of about 200 points. Note however that one can find restricted local areas in parameter space where the fine tuning is less severe than the above quoted average.

For the *spinorial representations* we find that the bottom KK tower plays an important role in electroweak symmetry breaking. Without including the bottom tower we could not

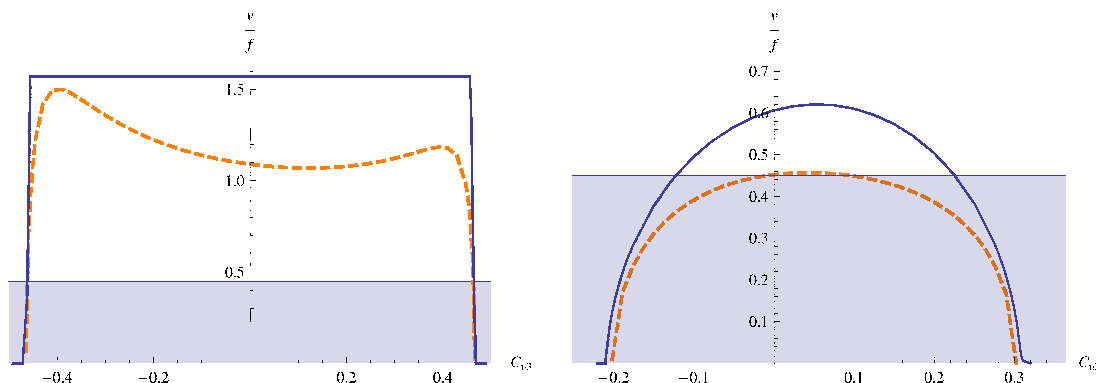


Figure 4: The plots show the dependence of \tilde{v}/f_π on c_u of the third generation for two sets of parameters in the model with fundamentals and adjoints. The first example on the left shows a generic point where there are only two narrow regions of c_u that lead to an acceptable electroweak breaking vacuum. The continuous line shows the result of the minimization of the full potential and the dashed line shows our approximation for regions where $\sin \tilde{v}/f_\pi$ is small. In the left plot for $-0.46 \lesssim c_{u_3} \lesssim 0.46$ the minimum of the potential is at $\tilde{v} = \pi f_\pi/2$, while for $c_{u_3} \lesssim -0.47$ and $c_{u_3} \gtrsim 0.47$ the minimum is at $\tilde{v} = 0$. In the case of $\tilde{v} = \pi f_\pi/2$, electroweak symmetry is maximally broken $m_W = gf/2$, whereas for $\tilde{v} = 0$, electroweak symmetry is unbroken ($m_W = 0$). In both cases there are massless fermions in the spectrum. For $-0.47 \lesssim c_{u_3} \lesssim -0.46$ and $0.46 \lesssim c_{u_3} \lesssim 0.47$ there is a minimum at intermediate values of \tilde{v}/f_π , yet an additional tuning is required to arrive at $\tilde{v}/f_\pi \ll 1$. For $\tilde{v}/f_\pi < 0.45$ we find $-0.464 < c_{u_3} < -0.462$ and $0.464 < c_{u_3} < 0.466$. In this case one needs to tune c_u to more than a percent level to get successful electroweak symmetry breaking. The right plot shows the same for carefully chosen values of the input parameters, where the local tuning is more modest. In this best case scenario we can have a realistic electroweak symmetry breaking minimum for the region $-0.21 < c_{u_3} < -0.13$ or $0.22 < c_{u_3} < 0.31$. The parameters chosen for this plot are $R'/R = 10^{16}$, $1/R' = 1.5$ TeV and $c_{Q_3} = 0.42$, $c_{d_3} = -0.56$, $\tilde{m}_u = 5$, $\tilde{m}_d = 1$, $\tilde{M}_u = 0$, $r = 1$ (right: $c_{Q_3} = 0.1$, $c_{d_3} = -0.56$, $\tilde{m}_u = 1$, $\tilde{M}_u = -0.5$, $r = 0.47$).

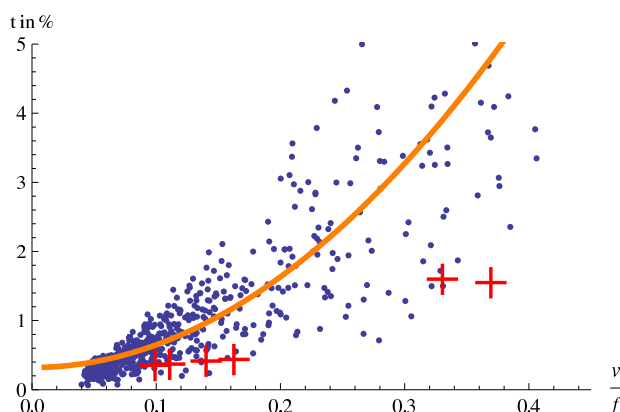


Figure 5: This plot shows the local tuning t for the model with adjoints and fundamentals. The orange line is a fit to a quadratic polynomial for which we find $t \sim \frac{1}{4}\epsilon^2$.

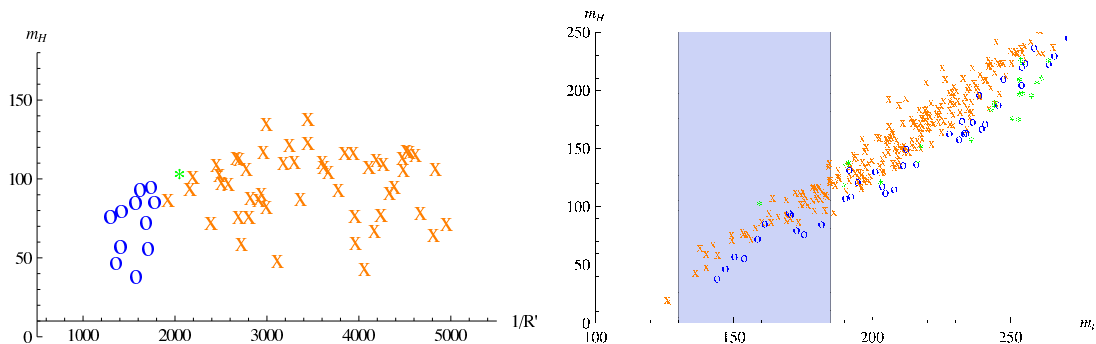


Figure 6: Left panel: the dependence of the Higgs mass on $1/R'$ for a variety of input parameters in the model with spinors that give successful electroweak symmetry breaking with points selected to give the top mass in the physical range. Right panel: the correlation between the top and the Higgs masses. Orange (x), blue (o), and green (*) points correspond to $\epsilon < 0.3$, $0.3 < \epsilon < 0.4$, and $0.4 < \epsilon$.

find any point with a sufficiently heavy Higgs mass. Including the bottom improves the situation because for the spinorial representation there is also a light KK mode in the bottom sector.⁷ However, it is exactly this state that is responsible for the large shift in the $Zb\bar{b}$ vertex. The fine tuning in the top sector is somewhat stronger for the spinorial case than for the fundamental+adjoint discussed above. Once we impose the fine-tuning in the input parameters of the theory, we still need to make sure that the Higgs is sufficiently heavy. Most of the time the top is too heavy. The reason behind this is that there is a very strong correlation between the Higgs and the top masses in this model. These correlations between the Higgs mass and $1/R'$ and m_{top} are summarized in figure 6.

4.1 Constraints from electroweak precision tests

Apart from yielding the correct electroweak breaking vacuum, phenomenologically acceptable GHU models must pass stringent electroweak precision tests [35, 27]. Since $SO(5)$ models are endowed with custodial symmetry, there is no tree-level constraints from the T parameter. The S parameter however is an issue. The expression for the S parameter (for vanishing brane kinetic terms) is given by [17]

$$S = 4\pi v^2 \left[\frac{(\int_R^{R'} a)(\int_R^{R'} a^{-1})^2 - \int_R^{R'} a(\int_z^{R'} a^{-1})^2}{\int_R^{R'} a^{-1}} \right] \approx \frac{3\pi v^2 R'^2}{2} \quad (4.25)$$

Demanding $S < .2$ yields the bound $R'^{-1} > 1.2 \text{ TeV}$. This implies that the lightest gauge boson KK modes have masses $\sim 2.4R'^{-1} \sim 3 \text{ TeV}$.

Another potentially dangerous contributions to the electroweak observables are the corrections to the $Zb_L\bar{b}_L$ coupling that is measured with the .25% accuracy. There are two potential sources of deviations in the $Zb\bar{b}$ couplings: mixing of the zero mode fermions

⁷We thank Roberto Contino for discussions on this point.

(after EWSB) with KK states of equal electric but different $SU(2)_L \times U(1)$ charges, and mixing among the gauge bosons.

We first estimate the contribution of the fermion mixing effect after EW breaking. Since the b gains a mass after EWSB, in principle one can no longer use the zero mode wave functions. Most of the time, these corrections are of order $(m_b R')^2$, and so can be safely ignored. However, in the spinorial model one encounters order $(m_t R')^2$ corrections. The leading effect of EWSB will be to twist the zero mode wave functions between the L and R multiplets of a bulk spinor via the Wilson line matrix. The effect of this twisting (for simplicity, we set $\tilde{M}_u = \tilde{M}_d = 0$) will be that the left handed bottom quark will end up partly living in the down-type component of $\chi_{b_u^c}$. The reason why there is a component in $\chi_{b_u^c}$ (but not in $\chi_{b_q^c}$ or $\chi_{b_d^c}$) is that the UV brane boundary conditions only allow a non-vanishing component in this mode. This non-vanishing component in $\chi_{b_u^c}$ will give the leading correction to the $Zb\bar{b}$ vertex. The coupling of the zero mode of Z (in the limit it is flat) is given by $g_{Zf\bar{f}} \sim (T_{3L} - \sin^2 \theta Q)$. Since the electric charges of all components that mix are the same the only correction comes from the deviation in the coupling to T_{3L} , which in our case is due to the fact that the $\chi_{b_u^c}$ component couples to T_{3R} and not to T_{3L} . So the relative deviation can be estimated to be (where $s^2 = 1 - c^2 = \sin^2 \frac{v}{2f}$).

$$\frac{\delta g_{Zb_L b_L}}{g_{Zb_L b_L}} \sim \frac{\sin^2(\tilde{v}/2f) \tilde{m}_u^2}{f_u^2 \left[\frac{1}{f_d^2} + \frac{\tilde{m}_u^2}{f_u^2} + \frac{\tilde{m}_d^2}{f_d^2} \right]} \quad (4.26)$$

This expression can be related to the formula for the top mass, and so we find

$$\frac{\delta g_{Zb\bar{b}}}{g_{Zb\bar{b}}} \sim \frac{(m_{\text{top}} R')^2}{f_u^2 f_{-u}^2} \sim \frac{(m_{\text{top}} R')^2}{(1 - 4c_u^2)} \quad (4.27)$$

For the range of interest $R' \sim 1 - 2 \text{ TeV}$ and $|c_u| < 1/2$ we will find a correction that is always at least a percent, and cannot be removed. For the other two models the effect does not occur due to the custodial protection proposed in ref. [18].

We move to the gauge contribution to the Zbb vertex. That correction can be calculated from the formula

$$\frac{\delta g_{Zb_L b_L}}{g_{Zb_L b_L}} = m_Z^2 \int_R^{R'} (z/R)^{-2c_q} \left(- \int_R^z z' \log(z'/R) + \frac{T_R^3 - \frac{g^2}{g'^2} Y}{T_L^3 - \frac{g^2}{g'^2} Y} \left(\int_R^z z' \log(R'/R) \right) \right) \quad (4.28)$$

We know from the analysis of the Higgs potential that we need $c_q < 1/2$. For generic left and right quantum numbers of b_L , the result is of order $m_Z^2 R'^2 \log(R'/R)$, that is enhanced by the large logarithm. This would lead to more stringent bounds than those from the S parameter. However, in the models where b_L is embedded in the bifundamental representation with $T_L^3 = T_R^3$ there is a cancellation of the large logs and the result is given by

$$\frac{\delta g_{Zb_L b_L}}{g_{Zb_L b_L}} \approx \frac{m_Z^2 R'^2}{4} \left(1 - \frac{4}{(3 - 2c_q)^2} \right) \quad (4.29)$$

This is below the experimental sensitivity even for $R' = 1 \text{ TeV}$.

5. Flavor in GHU

We begin our study of the flavor structure of GHU models. In this section we present a detailed discussion for the model of section 3.2, with each SM generation embedded in two fundamental and one adjoint SO(5) multiplet. The other two models lead to a very similar flavor structure, and we will later comment on the differences.

5.1 Fermion masses and mixing

We start by discussing the zero modes before EW breaking produces their mass terms. One difference with respect to the standard RS scenario is the presence of the IR boundary mass terms. These do not give masses to the zero modes, but imply that the zero modes are embedded in several bulk multiplets. In particular, the zero mode quark doublets q_L are embedded in all $\Psi_{q,u,d}$ in eq. (3.10), while the up-type quark singlets u_R lives in $\Psi_{q,u}$. On the other hand, the down-type quark singlets d_R lives only in Ψ_d . We write

$$q_q(x, z) \rightarrow \chi_{q_q}(z)q_L(x) \quad q_q(x, z) \rightarrow \chi_{q_u}(z)q_L(x) \quad q_d(x, z) \rightarrow \chi_{q_d}(z)q_L(x) \quad (5.1)$$

$$u_u^c(x, z) \rightarrow \psi_{u_u^c}(z)u_R(x) \quad u_q^c(x, z) \rightarrow \psi_{u_q^c}(z)u_R(x) \quad d_r(x, z) \rightarrow \psi_{d_r}(z)d_R(x) \quad (5.2)$$

Recall that we drop the generation index, and q_L, u_R, d_R are understood to be three-vectors in the generation space. The zero-mode profiles $\chi(z), \psi(z)$ are 3x3 matrices $\chi_q = \text{diag}(\chi_{q_1}, \chi_{q_2}, \chi_{q_3})$ that determine how much of the zero mode resides in each 5D fermion. Solving the equations of motion and the boundary conditions we find the left-handed profiles:

$$\begin{aligned} \chi_{q_q}(z) &= \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c_q} f_q \\ \chi_{q_u}(z) &= \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c_u} \tilde{m}_u^\dagger f_q \\ \chi_{q_d}(z) &= \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c_d} \tilde{m}_d^\dagger f_q \end{aligned} \quad (5.3)$$

where $f_q = \text{diag}(f(c_{q_1}), f(c_{q_2}), f(c_{q_3}))$ and $f(c)$ were defined in eq. (2.6). Similarly, the zero mode profiles for the right handed up-type fields are given by

$$\begin{aligned} \psi_{u_u^c}(z) &= \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{c_u} f_{-u} \\ \psi_{u_q^c}(z) &= -\frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{c_q} \tilde{M}_u f_{-u}, \end{aligned} \quad (5.4)$$

Finally, the down-type zero modes are contained only in the adjoints:

$$\psi_{d_r}(z) = \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{c_d} f_{-d}. \quad (5.5)$$

The overall normalization has been chosen such that we recover the usual normalized zero modes eq. (2.5) in the limit when the boundary masses are set to zero. In this form, our profiles closely resemble the corresponding formulae in the standard RS set-up. However,

in this basis, the kinetic terms for the zero modes are not diagonal. This kinetic mixing is parameterized by the Hermitian 3 by 3 matrices

$$\begin{aligned} K_q &= 1 + f_q \tilde{m}_u f_u^{-2} \tilde{m}_u^\dagger f_q + f_q \tilde{m}_d f_d^{-2} \tilde{m}_d^\dagger f_q, \\ K_u &= 1 + f_{-u} \tilde{M}_u^\dagger f_{-q}^{-2} \tilde{M}_u f_{-u}, \\ K_d &= 1, \end{aligned} \tag{5.6}$$

For example, for the doublet zero modes the kinetic term is given by $i\bar{q}_L(x)K_q\partial q_L(x)$. The kinetic mixing is inevitable in GHU models: since all the SM flavor mixing must originate from non-diagonal terms in the boundary mass terms, at least the matrix K_q must be non-diagonal. *This is an important difference with respect to the original RS set-up that will introduce additional contributions to flavor-violating processes.*

In GHU, the fermion masses originate from the bulk kinetic terms $\bar{\Psi}iD^M\Gamma_M\Psi$, where $D_z \rightarrow \partial_z - ig_5 A_z^a T^a$ and T^a are the SO(5) generators appropriate for a given representation. When A_z acquires a vev it produces a mass term connecting the quarks living in the same SO(5) multiplet. For the fundamental representation, the Wilson line marries the two up quarks in the bifundamental to the singlet up quark:

$$\left(\frac{R}{z}\right)^4 \frac{g_* v}{2} \frac{\sqrt{2}z}{R'} \bar{u}^c(u - \tilde{u}) \tag{5.7}$$

while for the adjoint representation, it couples the triplets to the bifundamental, for example

$$\left(\frac{R}{z}\right)^4 \frac{g_* v}{2} \frac{\sqrt{2}z}{R'} (\bar{d}_l - \bar{d}_r) d_d \tag{5.8}$$

Plugging in the zero mode profiles we find for the mass matrix (in the basis where the kinetic terms are not diagonal)

$$\begin{aligned} m_u &= \frac{g_* v}{2\sqrt{2}} f_q (\tilde{m}_u - \tilde{M}_u) f_{-u} \\ m_d &= \frac{g_* v}{2\sqrt{2}} f_q \tilde{m}_d f_{-d} \end{aligned} \tag{5.9}$$

To obtain the actual masses and mixing angles one needs to first diagonalize the kinetic mixing terms and rescale the fields. We decompose $K_a = V_a N_a V_a^\dagger$ for $a = q, u, d$, where N is a positive diagonal matrix and V a unitary matrix, and we define the corresponding Hermitian matrix $H_a = V_a N_a^{-1/2} V_a^\dagger$. The Hermitian rotation of the zero modes, $q_L \rightarrow H_q q_L$, $u_R \rightarrow H_u u_R$, brings the kinetic terms to the canonical form. The mass matrices rotate into

$$\begin{aligned} m_u^{\text{SM}} &= \frac{g_* v}{2\sqrt{2}} H_q f_q (\tilde{m}_u - \tilde{M}_u) f_{-u} H_u \\ m_d^{\text{SM}} &= \frac{g_* v}{2\sqrt{2}} H_q f_q \tilde{m}_d f_{-d}. \end{aligned} \tag{5.10}$$

In the next step we decompose, as usual, the up and down mass matrices as $m_{u,d}^{\text{SM}} = U_{L,u,d} m_{u,d} U_{R,u,d}^\dagger$, and we perform a unitary rotation of the zero mode quarks so as to diagonalize the mass matrix, $d_{L,R} \rightarrow U_{L,R} d_{L,R}$, $u_{L,R} \rightarrow U_{L,R} u_{L,R}$.

The flavor structure following from 5.10 is very similar to that of the ordinary RS model with anarchic flavor structure, cf. (2.8). The main difference between (5.10) and (2.8) is the appearance of the extra Hermitian matrices H_a that originate from the kinetic mixing. Note that, since N_a the eigenvalues of the kinetic mixing matrices (5.6), all entries of N_a are very close to 1 except maybe for the third generation. For this reason, the hierarchical structure of the mass matrix will turn out to be very similar to that in RS.

The quark masses that follow from (5.10) are as usual, approximately equal to the diagonal elements. They can be estimated by

$$\begin{aligned}
 m_u &\sim \frac{g_* v}{2\sqrt{2}} \frac{(\tilde{m}_u - \tilde{M}) f_q f_{-u}}{\sqrt{\left(1 + \frac{f_q^2 \tilde{m}_u^2}{f_u^2} + \frac{f_q^2 \tilde{m}_d^2}{f_d^2}\right) \left(1 + \frac{f_{-u}^2 \tilde{M}^2}{f_{-q}^2}\right)}} \\
 m_d &\sim \frac{g_* v}{2\sqrt{2}} \frac{\tilde{m}_d f_q f_{-d}}{\sqrt{\left(1 + \frac{f_q^2 \tilde{m}_u^2}{f_u^2} + \frac{f_q^2 \tilde{m}_d^2}{f_d^2}\right)}}
 \end{aligned}
 \tag{5.11}$$

where $\tilde{m}_{u,d}, \tilde{M}$ here denotes the typical amplitude of the entries in the corresponding mass matrix. As in the standard RS, the mass hierarchies are set by f_q, f_{-u}, f_{-d} and $g_*/2$ plays the similar role to the Yukawa coupling Y_* . The coupling g_* is however related to the experimentally measured weak coupling, see eq. (3.2), and cannot be varied, unless we consider large UV brane kinetic terms. Another difference is the dependence on the boundary masses that enters both the numerator and the denominator. The latter may be significantly different than 1 only when $f_x \sim 1$, which is the case for the third generation. In that case $\tilde{m}_{u,d}, \tilde{M}$ saturate: further increasing it does not increase the mass.

We turn to discussing the mixing angles and how are they affected by the kinetic mixing. Consider the doublet mixing matrix K_q . f_q is hierarchical, $f_{q_1} \ll f_{q_2} \ll 1, f_{q_3} \sim 1$, while $f_{u,d}$'s are all of order one (since it is $f_{-u,d}$ that sets the mass hierarchy). Thus, $K_q \sim \delta_{ij} + \tilde{m}^2 f_{q_i} f_{q_j}$, where we denote $\tilde{m}^2 = \tilde{m}_u^2 + \tilde{m}_d^2$. It follows that $N_q \sim (1, 1, 1 + \tilde{m}^2)$ and $(V_q)_{12} \sim \tilde{m}^2 f_{q_1} f_{q_2}, (V_q)_{13} \sim \tilde{m}^2 f_{q_1} / (1 + \tilde{m}^2), (V_q)_{23} \sim \tilde{m}^2 f_{q_2} / (1 + \tilde{m}^2)$. At the end of the day, the left hierarchy is set by the matrix $f_q H_q$ (rather than f_q as in RS) whose diagonal elements are of the form

$$H_q f_q \sim \begin{pmatrix} f_{q_1} & & \\ & f_{q_2} & \\ & & f_{q_3} (1 + f_{q_3}^2 \tilde{m}^2)^{-1/2} \end{pmatrix},
 \tag{5.12}$$

The off-diagonal terms in the above matrix are irrelevant for the following discussion. We can see that the corrections coming from the kinetic mixing do not change the hierarchy of f_q . There are, of course, $\mathcal{O}(1)$ corrections in the actual numerical values of the f 's that are required to reproduce the mass and mixing hierarchies, but their orders of magnitude are unchanged so that implementation of flavor hierarchies is analogous as in RS. The only parametric difference between f_q and eq. (5.12) is that the third eigenvalue gets suppressed by $1/\tilde{m}$ as soon as \tilde{m} is larger than 1. We keep track of that effect because, as the study of the Higgs potential shows, the interesting parameter space with successful electroweak

breaking and the large enough top quark mass extends to $\tilde{m} \sim \text{few}$. This parametric dependence feeds into the left rotation that diagonalize the SM mass matrix,

$$(U_{Lu,d})_{12} \sim \frac{f_{q_1}}{f_{q_2}} \quad (U_{Lu,d})_{13} \sim \frac{f_{q_1}}{f_{q_3}}(1 + f_{q_3}^2 \tilde{m}^2)^{1/2} \quad (U_{Lu,d})_{23} \sim \frac{f_{q_2}}{f_{q_3}}(1 + f_{q_3}^2 \tilde{m}^2)^{1/2} \quad (5.13)$$

The consequence is that the relation between f_q and the CKM angles is slightly modified, $(1 + \tilde{m}^2)^{1/2} f_{q_1} \sim \lambda^3$, $(1 + \tilde{m}^2)^{1/2} f_{q_1} \sim \lambda^2$, where λ is the Cabibbo angle.

By the same token,

$$H_u f_{-u} \sim \begin{pmatrix} f_{-u_1} & & \\ & f_{-u_2} & \\ & & f_{-u_3}(1 + f_{-u_3}^2 \tilde{M}^2)^{-1/2} \end{pmatrix}, \quad (5.14)$$

and the elements of the right rotation matrix for the up-type quark are

$$(U_{Ru})_{12} \sim \frac{f_{-u_1}}{f_{-u_2}} \quad (U_{Ru})_{13} \sim \frac{f_{-u_1}}{f_{-u_3}}(1 + f_{-u_3}^2 \tilde{M}^2)^{1/2} \quad (U_{Ru})_{23} \sim \frac{f_{-u_2}}{f_{-u_3}}(1 + f_{-u_3}^2 \tilde{M}^2)^{1/2} \quad (5.15)$$

Finally, the elements of the right rotation matrix for the down-type quark are not affected by the kinetic mixing,

$$(U_{Rd})_{12} \sim f_{-d_1}/f_{-d_2} \quad (U_{Rd})_{13} \sim f_{-d_1}/f_{-d_3} \quad (U_{Rd})_{23} \sim f_{-d_2}/f_{-d_3} \quad (5.16)$$

5.2 Flavor constraints

We are ready to evaluate the flavor constraints in the GHU models that originate from a tree-level exchange of the KK gluons. To this end, we need to compute the couplings of the SM down-type quarks to the lightest KK gluon,

$$g_L^{ij} \bar{d}_L^i \gamma_\mu G_{\mu(1)} d_L^j + g_R^{ij} \bar{d}_R^i \gamma_\mu G_{\mu(1)} d_R^j \quad (5.17)$$

As before, we introduce the diagonal matrix $g_x \approx g_{s*}(-\frac{1}{\log R'/R} + \gamma(c_x) f_x^2)$, $x = q, -u, -d$, that approximate the couplings of quarks in each 5D multiplets to the lightest KK gluon. The complication inherent to the GHU models is that the zero mode quarks are contained in several 5D multiplets. This is already one source of off-diagonal couplings. Furthermore, on top of the unitary rotation that diagonalizes the SM mass matrix, the off-diagonal terms are affected by the Hermitian rotation that diagonalizes the kinetic terms. At the end of the day the couplings can be written as

$$g_L^{ij} \approx \left[U_L^\dagger d H_q \left(g_q + f_q \tilde{m}_u f_u^{-2} g_u \tilde{m}_u^\dagger f_q + f_q \tilde{m}_d f_d^{-2} g_d \tilde{m}_d^\dagger f_q \right) H_q U_L d \right]_{ij}$$

$$g_R^{ij} \approx \left[U_R^\dagger d g_{-d} U_R d \right]_{ij} \quad (5.18)$$

Compared to the RS formula (2.16), the doublet quark off-diagonal couplings receive many additional contributions. In spite of that, the RS-GIM mechanism is still at work, in the

sense that the off-diagonal terms are always multiplied by the hierarchical matrix f_q . The flavor-changing couplings are proportional to

$$\begin{aligned}
 g_L^{ds} &\sim g_{s*}(1 + \tilde{m}^2)f_{q_1}f_{q_2} \\
 g_L^{db} &\sim g_{s*}\frac{1 + \tilde{m}^2}{(1 + f_{q_3}^2\tilde{m}^2)^{1/2}}f_{q_1}f_{q_3} \\
 g_L^{sb} &\sim g_{s*}\frac{1 + \tilde{m}^2}{(1 + f_{q_3}^2\tilde{m}^2)^{1/2}}f_{q_2}f_{q_3} \\
 (g_{R,d})_{ij} &\sim g_{s*}f_{-d_i}f_{-d_j}
 \end{aligned}
 \tag{5.19}$$

We can now estimate the size of the FCNC four-fermion operators relevant for the for the Kaon mixing. The LL operator is, just like in RS, suppressed by the CKM matrix elements,

$$C_K^1 \sim \frac{1}{6M_G^2}g_{s*}^2|V_{ts}^*V_{td}|^2f_{q_3}^4\frac{(1 + \tilde{m}^2)^2}{(1 + f_{q_3}^2\tilde{m}^2)^2}
 \tag{5.20}$$

This bound has almost the same form as in the RS case, except for the last factor that depends on \tilde{m} . In RS one usually takes $f_{q_3} \sim 0.3$ (that is c_{q_3} close to one half) to further suppress the LL operator. In the case at hand, using the approximate expression for the top mass, we can relate (for $f_{-u_3} \sim 1$)

$$f_{q_3}\sqrt{\frac{1 + \tilde{m}^2}{1 + f_{q_3}^2\tilde{m}^2}} \sim \frac{2\sqrt{2}m_t}{g_*v}\frac{\sqrt{1 + \tilde{m}^2}\sqrt{1 + \tilde{M}^2}}{\tilde{m}_u - \tilde{M}}
 \tag{5.21}$$

Taking $\tilde{m}_{u,d} \sim \tilde{M} \sim 1$ we can rewrite our estimate as

$$C_K^1 \sim \frac{1}{(5 \cdot 10^4 \text{ TeV})^2} \left(\frac{3 \text{ TeV}}{M_G} \right)^2
 \tag{5.22}$$

which shows that a 3 TeV KK gluon satisfies the bound listed in table 2.

On the other hand, the LR contribution is

$$C_K^4 = -3C_K^5 \sim \frac{1}{M_G^2}\frac{g_{s*}^2}{g_*^2}\frac{8m_dm_s}{v^2}\frac{1 + \tilde{m}^2}{\tilde{m}_d^2}
 \tag{5.23}$$

where this time we related f 's to down-type quark masses via $f_{q_i}f_{-d_i} \sim 2\sqrt{2}m_d^i/g_*v\tilde{m}_d$. Again, this estimate is very similar as in the RS case, with Y_* replaced by $g_*/2$. The new parametric difference is the factor $(1 + \tilde{m}^2)/\tilde{m}_d^2$, which can be traced back to the kinetic mixing. This factor may enhance (but never suppress) the coefficient, if $\tilde{m}_d < 1$ or $\tilde{m}_u > \tilde{m}_d$. In consequence (when boundary kinetic terms are ignored),

$$C_K^4 \sim \frac{1}{(1.5 \cdot 10^4 \text{ TeV})^2} \left(\frac{3 \text{ TeV}}{M_G} \right)^2
 \tag{5.24}$$

The numbers refer to the case when the boundary kinetic terms at the TeV scale are small, so that $g_* \sim g \log^{1/2}(R'/R) \sim 4$, $g_{s*} \sim g_s \log^{1/2}(R'/R) \sim 6$. We can see that $\Lambda_{ImC_K^4}$ is a

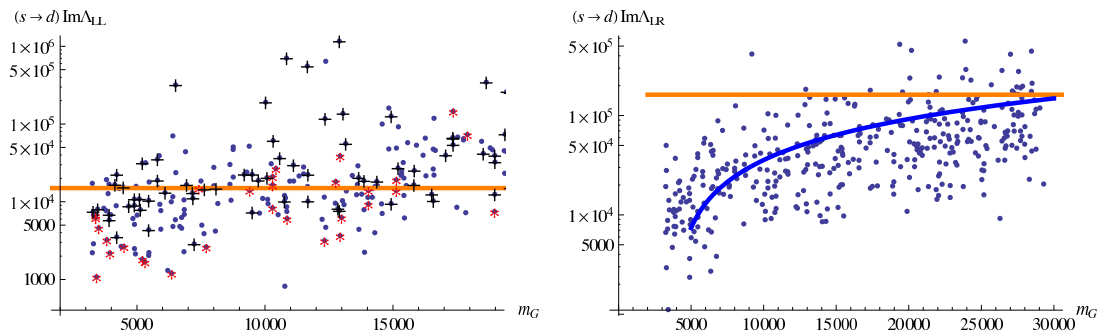


Figure 7: Scan of the effective suppression scale of $\text{Im}C_{1K}$ (left panel) $\text{Im}C_{4K}$ (right panel) and in the scenario with adjoint and fundamental bulk fields. In the left panel points with $\tilde{m}_u > 3.5$ are red (*) and points with $\tilde{m}_u < 1.5$ are black (+). All the points give the correct low-energy spectrum but most of the points with $m_G < 30$ TeV fail to satisfy the $\text{Im}C_{4K}$ bound of $\Lambda > 1.6 \cdot 10^5$ TeV. The blue line is a linear fit of the M_G dependence.

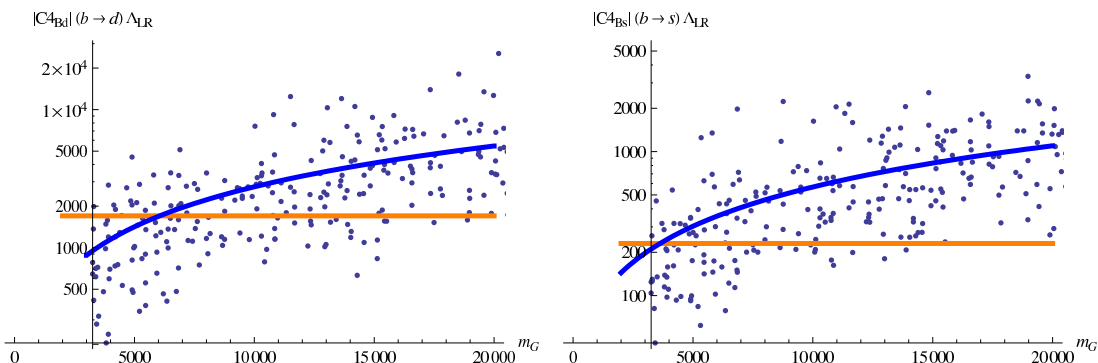


Figure 8: Scan of the effective suppression scale of $|C_{B_d}^4|$ (left panel) $|C_{B_s}^4|$ (right panel) and in the scenario with adjoint and fundamental bulk fields. All the points give the correct low-energy spectrum and points with $m_G > 5$ TeV mostly satisfy the bounds. The blue lines are linear fits of the M_G dependence.

factor of ~ 11 too small for a 3 TeV KK gluon. To satisfy the bound $\Lambda_{\text{Im}C_K^4} > 16 \cdot 10^4$ TeV (see table 2) would require a KK scale of about 33 TeV.

As in RS, we can change this estimate by playing with the boundary kinetic terms for $\text{SU}(2)_L$ and $\text{SU}(3)_C$. Assuming a large bare UV coupling (small brane kinetic term) for $\text{SU}(3)_c$ and including the one-loop QCD running we can decrease g_{s*} , thus relaxing the bound by a factor of two. On the other hand, we could make the coupling g_* larger by adding a large brane kinetic term for $\text{SU}(2)_L$, that is to pick $r > 1$. In the best of all worlds, assuming $g_{s*} \sim 3$ and $g_* \sim 4\pi$ (the latter implies lack of perturbative control over the Higgs sector) we could lower the bound down to 5 TeV. In a calculable framework it is not possible to lower the bound below 10 TeV. These estimates demonstrate that the GHU framework with fully anarchic flavor is not a plausible scenario.

5.3 Numerical scan

We have verified the above estimates of the flavor bounds by an extensive numerical scan over the parameter space of the model. Our first aim is to find realistic electro-weak symmetry breaking minima of the effective potential. We are only interested in points with a sizable gap between the EW scale and the scale of the KK modes ($v/f_\pi \lesssim 0.3$). The parameters related by EWSB are: the bulk masses of the third generation $c_{q_3}, c_{u_3}, c_{d_3}$, the size of the brane masses $\tilde{m}_u, \tilde{m}_d, \tilde{M}_u, \tilde{M}_d$, the KK scale $1/R'$. We keep the hierarchy $R/R' = 10^{-16}$ fixed. Contrary to the RS case, we are not free to pick any combination, since most choices do not lead to EWSB satisfying our criteria. As we have discussed, v/f_π is very sensitive to q_{q_3}, c_{u_3} , see e.g. figure 4. We therefore first randomly generate a set of parameters with $c_{q_3} \in [0.2, 0.48]$, $1/R' \in [900, 12500]$ GeV, $c_{-d_3} = -0.55$, $|\tilde{m}_u| \in [0.5, 5]$, $|\tilde{m}_d| \in [0.5, 2]$, and $r \in [0, 0.8]$. (For simplicity, we have set $\tilde{M}_u = \tilde{M}_d = 0$). Knowing R, R' and r , we determine v/f_π from the condition to reproduce m_W , see (3.5). We then find a c_{u_3} such that the minimum of the potential really is at the value of v/f_π specified before. Finally, we want the theory to be calculable, which puts an upper bound on the bulk SO(5) coupling g_* . This can be equivalently expressed as a bound on the number of colors of the dual CFT, see eq. (3.1), and we require $N_{\text{CFT}} \gtrsim 5$.

The next step is to calculate the SM masses, mixing angles and KK FCNCs. Given the size of $c_{q_3}, c_{u_3}, c_{d_3}$ and of \tilde{m}_u and \tilde{m}_d we can determine the bulk masses of the first two generations. First, c_{q_1} and c_{q_2} are fixed since left rotations (5.13) need to have the same hierarchy as the CKM. The remaining bulk masses $c_{u_{1,2}}, c_{d_{1,2}}$ can be fixed using (2.13) requiring the mass eigenvalues to match the SM at TeV scales.

Now we randomly generate complex 3×3 matrices \tilde{m}_u, \tilde{m}_d with eigenvalues approximately of the size as those used in the calculation of the potential above. Using (5.10) we then calculate the effective mass matrices $m_{u,d}^{\text{SM}}$. These mass matrices only approximately reproduce the SM. In order to have a completely realistic set of parameters we need to parameterize the deviation from the SM and reject those $\tilde{m}_{u,d}$ which deviate too much. For this purpose we have defined matrix norms that measure the distance between the generated and the physical values at 3 TeV. We calculate the distance between

- The mass eigenvalues to the the SM running masses at 3 TeV, see table. 1.
- The moduli of the generated CKM to

$$|V_{\text{CKM}}| \approx \begin{pmatrix} 0.97 & 0.226 & 0.0035 \\ 0.226 & 0.97 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix}, \quad (5.25)$$

which we took from a recent tree level determination [36].

- The amount of CP violation to the SM Jarlskog invariant [37]

$$J \approx 3 \cdot 10^{-5} \quad (5.26)$$

We finally accept only those random matrices $\tilde{m}_{u,d}$ for which the maximal relative distance of any constraint is below 30%.

The results of the scan are displayed in figure 7. Most of the points with $m_G < 30$ TeV fail to satisfy the ImC_{4K} bound of $\Lambda > 1.6 \cdot 10^5$ TeV, in agreement with our analytical estimates. Just like in the RS case this bound is somewhat dependent on the assumption that one makes on the boundary kinetic terms for the gluon. For the two extreme cases the numerical values for the suppression scales are the following. In the worst case when the theory is barely perturbative, the bound is enhanced to about 54 TeV, while in the best case, with no bare brane kinetic terms to bound is reduced to about 17 TeV. We conclude that the flavor constraints on the warped models with a pseudo-Goldstone Higgs are even stronger than that in the standard RS with a TeV brane localized Higgs.

In the remainder of this section we summarize the flavor constraints for the other two GHU models considered in this paper. The short summary is that in none of the models is the RS flavor bound relaxed, but rather strengthened.

5.4 Fermions in the spinorial

For the model with each generation of the SM fermions embedded in 3 SO(5) spinors the parameter space includes an additional boundary mass matrix \tilde{M}_d that introduces the kinetic mixing also to the d -singlet quark sector. The kinetic mixing matrices are given by

$$\begin{aligned} K_q &= 1 + f_q \tilde{m}_u f_u^{-2} \tilde{m}_u^\dagger f_q + f_q \tilde{m}_d f_d^{-2} \tilde{m}_d^\dagger f_q, \\ K_u &= 1 + f_{-u} \tilde{M}_u^\dagger f_{-q}^{-2} \tilde{M}_u f_{-u}, \\ K_d &= 1 + f_{-d} \tilde{M}_d^\dagger f_{-q}^{-2} \tilde{M}_d f_{-d}, \end{aligned} \tag{5.27}$$

The SM up and down mass matrix take the form

$$m_{u,d}^{\text{SM}} = \frac{g_* v}{4} H_q f_q (\tilde{m}_{u,d} - \tilde{M}_{u,d}) f_{-u,d} H_{u,d}. \tag{5.28}$$

There is an additional factor of $1/\sqrt{2}$ that is a group theoretical factor of the spinorial SO(5) representation. The kinetic mixing induced by \tilde{M}_d feeds into the right-rotation unitary matrix for the down quarks

$$(U_{Rd})_{12} \sim \frac{f_{-d_1}}{f_{-d_2}} \quad (U_{Rd})_{13} \sim \frac{f_{-d_1}}{f_{-d_3}} (1 + f_{-d_3}^2 \tilde{M}^2)^{1/2} \quad (U_{Rd})_{23} \sim \frac{f_{-d_2}}{f_{-d_3}} (1 + f_{-d_3}^2 \tilde{M}^2)^{1/2} \tag{5.29}$$

but the effect is negligible (unless \tilde{M} is very large) because $f_{-d_3} \ll 1$ to account for $m_b/m_t \ll 1$. The only difference in flavor constraints is the consequence of the $1/\sqrt{2}$ factor in the SM mass matrix, which makes the LR flavor bounds a factor of $\sqrt{2}$ more *stringent*.

5.5 Fermions in four fundamentals

The model with four fundamental representation per generation contains yet another matrix θ that is a source of flavor violation. The kinetic mixing matrices are given by

$$\begin{aligned} K_q &= 1 + f_{q_u} (R'/R)^{c_{q_u}} \theta^\dagger (R'/R)^{-c_{q_d}} f_{q_d}^{-2} (R'/R)^{-c_{q_d}} \theta (R'/R)^{c_{q_u}} f_{q_u} + f_{q_u} \tilde{m}_u^\dagger f_u^{-2} \tilde{m}_u f_{q_u} \\ &\quad + f_{q_u} (R'/R)^{c_{q_u}} \theta^\dagger (R'/R)^{-c_{q_d}} \tilde{m}_d f_d^{-2} \tilde{m}_d^\dagger (R'/R)^{-c_{q_d}} \theta (R'/R)^{c_{q_u}} f_{q_u}, \\ K_u &= 1 + f_{-u} \tilde{M}_u^\dagger f_{-q_u}^{-2} \tilde{M}_u f_{-u}, \\ K_d &= 1 + f_{-d} \tilde{M}_d^\dagger f_{-q_d}^{-2} \tilde{M}_d f_{-d}, \end{aligned} \tag{5.30}$$

while the mass terms are

$$\begin{aligned}
 m_u^{\text{SM}} &= \frac{g_* v}{2\sqrt{2}} H_q f_{q_u} (\tilde{m}_u - \tilde{M}_u) f_{-u} H_u \\
 m_d^{\text{SM}} &= \frac{g_* v}{2\sqrt{2}} H_q f_{q_u} (R'/R)^{c_{q_u}} \theta^\dagger (R'/R)^{-c_{q_d}} (\tilde{m}_d - \tilde{M}_d) f_{-d} H_d
 \end{aligned}
 \tag{5.31}$$

There are several new possible flavor effects here. First of all, the mixing matrix θ shows, and it is accompanied by the large factor $(R'/R)^{c_{q_u}}$ on the outside. This factor perfectly counteracts the hierarchical suppression generated by the matrix f_q . In consequence, as long as θ is an arbitrary anarchic matrix, we definitely lose the RS hierarchic structure of the fermion mass matrix. The origin of this is easy to understand. Here we have two copies of the left handed doublets, and we are removing one combination with an additional right handed doublet on the UV brane. If the matrix θ is anarchic, then we introduce a large flavor violating effect into the elementary sector, that is unsuppressed by the mixing of elementary and composite states.

In order for the UV physics to maintain an $SU(3)_Q$ flavor symmetry for the doublets we need to assume that θ is, to a good approximation, proportional to the unit matrix. In the following we set $\theta \rightarrow \Theta 1_{3 \times 3}$ in all expressions, where Θ is a c-number. This is in fact a similar assumption as the one implicitly makes in RS and the other two GHU models: the elementary sector is flavor symmetric (that is there are no large flavor violating kinetic mixing terms on the Planck brane), and only the CFT gives rise to flavor violations. With this ansatz, the flavor structure becomes very similar to that in the spinorial model. There is still another new effect, however, which is the presence of two sets of c_q . As a consequence, in the expression for $m_{u,d}^{\text{SM}}$ the left hierarchy is not set by the same functions: for the up-type masses it is set by $f_{c_{q_u}}$ while for the down-type by $f_{c_{q_u}} (R'/R)^{c_{q_u} - c_{q_d}}$. This effect can be used as an alternative way to explain the large isospin breaking in the third generation. Instead of taking $f_{-d_3} \ll 1$, that is assuming b_R is mostly elementary, we can choose $f_{-d_3} \sim 1$ and $c_{q_u^3} < c_{q_d^3}$ to obtain $m_b/m_t \ll 1$. Unfortunately, this new avenue does not seem to lead to suppressing dangerous four-fermion operators. The coefficients C_K^4 can be estimated as

$$C_K^4 \sim (R'/R)^{(c_{q_d^1} - c_{q_1}) + (c_{q_d^2} - c_{q_2})} \frac{1}{M_G^2} \frac{g_{s*}^2}{g_*^2} \frac{8m_d m_s}{v^2} \frac{1 + \tilde{m}^2}{\tilde{m}_d^2}
 \tag{5.32}$$

where $c_{q_i} = \min(c_{q_u^i}, c_{q_d^i})$. For $c_{q_u^{1,2}} < c_{q_d^{1,2}}$ we are able to enhance the coefficient C_K^4 , but it's not exactly what we want.

6. Conclusion

In this paper we studied flavor physics in the framework of 5D warped GHU models that provide a dual realization of a composite pseudo-Goldstone Higgs. This is an extension of the standard RS scenario, that makes the electroweak symmetry breaking dynamical and fully calculable.

The flavor structure of GHU models turns out to be quite similar to that in RS. The hierarchical structure of the quark masses and the CKM matrix appears as a consequence

of different localization of zero modes in the extra dimension. The RS-GIM mechanism is operating, in the sense that the coefficients of effective four-fermion $\Delta F = 2$ operators induced by the tree-level KK mode exchange are suppressed by the small off-diagonal CKM matrix elements (LL operators), or by the light quark masses (LR operators).

The GHU models introduce, however, new contributions to flavor violation that are of the same order of magnitude as those in RS. The reason is that the larger set of local symmetries (that is crucial to realize the Higgs as a pseudo-Goldstone boson) imply a different realization of the zero-mode fermionic sector. In particular, the zero modes must be embedded in more than one bulk multiplet. We call it the kinetic mixing because in the original flavor basis in 5D it shows up as the generation mixing via the zero modes kinetic terms. This effect is controlled by the IR boundary masses - the same Lagrangian parameters that control also the mass matrix. The kinetic mixing feeds into the effective four-fermion $\Delta F = 2$ operators, parametrically enhancing its coefficients by a factor of few.

Just like in RS, the strongest bound on the KK scale comes from the imaginary part of the LR $(sd)^2$ operator, that affects CP violation in the kaon sector. We find that for anarchic boundary mass matrices, the generic bounds on the lightest KK gluon mass is of order 30 TeV. Such a large KK scale of course undermines the motivations for the GHU models. This implies that GHU does not make sense without additional flavor symmetries. There are several ways one could suppress the dangerous FCNC contributions (apart from assuming accidental cancelations). For example in the models of [39] one imposes a bulk flavor symmetry, and all mixing originates from Planck brane localized kinetic mixing terms. As a result one can construct models where all tree-level FCNC's are absent (a genuine GIM mechanism), but of course in this case one gives up on the explanation of the fermion mass hierarchy. A recent proposal suggests to truncate the bulk at a scale of about 10^3 TeV which softens the flavor problem by effectively reducing g_{s*} [41]. The obvious price to pay is giving up on an explanation for the hierarchy between weak and Planck scale. An intermediate solution could be to try to construct flavor models that do explain both hierarchies, but have some partial flavor symmetries left over [25, 40]. One possible ansatz that would alleviate the constraints while still giving rise to the hierarchies would be to assume that the boundary masses in the down sector \tilde{m}_d, \tilde{M}_d are proportional to the unit matrix, so that all the CKM mixing originate from non-diagonal elements in \tilde{m}_u, \tilde{M}_u . This suppresses the right rotation matrix U_{Rd} , so that the LR operator is also suppressed.

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A. KK gluon sums

In this appendix we present analytical formulas that allow one to include the contribution of the entire KK gluon tower to the effective four-fermion operators. We start with couplings of the n -th KK gluon to the quark eigenstates

$$g_{L,n}^{ij} \bar{q}_L^i \gamma_\mu G^{\mu(n)} q_L^j + g_{R,n}^{ij} \bar{q}_R^i \gamma_\mu G^{\mu(n)} q_R^j \quad (\text{A.1})$$

The couplings are given by

$$g_{L,n}^{ij} = g_{s*} R^{1/2} \int_R^{R'} f_n(z) h_L^{ij}(z) \quad g_{R,n}^{ij} = g_{s*} R^{1/2} \int_R^{R'} f_n(z) h_R^{ij}(z) \quad (\text{A.2})$$

Above, $a(z)$ is the warp factor (that we keep arbitrary here), $f_n(z)$ is the profile of the n -th KK gluon, and g_5 is the dimensionful bulk coupling of SU(3) color. For vanishing brane kinetic terms, the SM strong coupling is given by $g_s^2 = g_{s*}^2 R/L$, where $L = \int_R^{R'} a(z)$. The quark bilinear profiles $h_{L,R}^{ij}(z)$ are 3x3 matrices in the mass eigenstate basis. They are related to the fermionic profiles:

$$h_L(z) = a^4(z) V_L^\dagger \sum_a \chi_{q_a}^\dagger(z) \chi_{q_a}(z) V_L \quad h_R(z) = a^4(z) V_R^\dagger \sum_a \psi_{q_a}^\dagger(z) \psi_{q_a}(z) V_R \quad (\text{A.3})$$

where the sum goes over all bulk multiplets in which the quark eigenstates is embedded, and $V_{L,R}$ collectively denote all Hermitian or unitary rotations that relate the original flavor basis to the mass eigenstate basis. By orthogonality, the bilinear profiles satisfy $\int_R^{R'} h_{L,R}^{ij} = \delta_{ij}$.

The coefficients of the effective four-fermion operators are set by the sums of the form

$$\Sigma_{LR}^{ij,kl} = \sum_{n=1}^{\infty} \frac{g_{L,n}^{ij} g_{R,n}^{kl}}{m_n^2} \quad (\text{A.4})$$

Strangely enough, with the help of the methods of ref. [42] the sum can be evaluated for a general warp factor (even though there is no closed expression for particular m_n and g_n). The strategy is to 1) insert the integral expression for the couplings into the sum, 2) use the integrated equation of motion for $f_n(z)$, and 3) use the completeness relation for $f_n(z)$. When the smoke clears, one is left with

$$\begin{aligned} \Sigma_{LR}^{ij,kl} = g_s^2 & \left[L \int_R^{R'} a^{-1}(z) \left(\int_R^z h_L^{ij}(z') \right) \left(\int_R^z h_R^{kl}(z') \right) \right. \\ & + \delta_{ij} \int_R^{R'} h_R^{kl}(z) \int_R^z a^{-1}(z') \int_R^{z'} a(z'') + \delta_{kl} \int_R^{R'} h_L^{ij}(z) \int_R^z a^{-1}(z') \int_R^{z'} a(z'') \left. \right] \\ & - g_s^2 \delta^{ij} \delta^{kl} \left[\int_R^{R'} a^{-1}(z') \int_R^{z'} a(z'') + L^{-1} \int_R^{R'} a(z) \int_R^z a^{-1}(z') \int_R^{z'} a(z'') \right] \end{aligned} \quad (\text{A.5})$$

The result is quite complicated, but it simplifies considerably for flavor changing sums, which are of primary interest here:

$$\Sigma_{LR}^{ij,kl} = g_s^2 L \int_R^{R'} a^{-1}(z) \left(\int_z^{R'} h_L^{ij}(z') \right) \left(\int_z^{R'} h_R^{kl}(z') \right) \quad i \neq j \quad k \neq l \quad (\text{A.6})$$

Thus, the whole sum is expressed by simple integrals of the warp factor and the fermion profiles. In particular, for AdS geometry we take $a(z) = R/z$, and the fermionic profiles from eq. (2.5). The inverse of the AdS warp factor is sharply peaked towards IR. Therefore the integrals depend mainly on the IR value of the fermionic profiles, that also sets the fermion masses and mixing angles. This is the origin of the RS-GIM mechanism.

The coefficients of the operators relevant for the kaon sector are given by

$$\begin{aligned}
 C_{1K} &= \frac{1}{6} g_s^2 \log(R'/R) \int_R^{R'} z \left(\int_z^{R'} h_L^{sd}(z') \right) \left(\int_z^{R'} h_L^{sd}(z') \right) \\
 C_{4K} &= -g_s^2 \log(R'/R) \int_R^{R'} z \left(\int_z^{R'} h_L^{sd}(z') \right) \left(\int_z^{R'} h_R^{sd}(z') \right) \\
 C_{5K} &= -\frac{1}{3} C_{4K}
 \end{aligned}
 \tag{A.7}$$

In RS, the bilinear profiles of the down-type quarks are given by

$$\begin{aligned}
 h_{L,d}(z) &= R'^{-1} U_{L,d}^\dagger \left(\frac{R'}{z} \right)^{2c_q} f_q^2 U_{L,d} \\
 h_{R,d}(z) &= R'^{-1} U_{R,d}^\dagger \left(\frac{R'}{z} \right)^{-2c_d} f_{-d}^2 U_{R,d}
 \end{aligned}
 \tag{A.8}$$

Here, the non-diagonal elements are only due to the left and right unitary rotations that diagonalize the SM mass matrix.

For the GHU model with two fundamentals and an adjoint, the right-handed down-quark bilinear profile remains the same, but the left-handed one becomes more complicated due to the kinetic mixing:

$$h_{L,d}(z) = R'^{-1} U_{L,d}^\dagger H_q \left[\left(\frac{R'}{z} \right)^{2c_q} f_q^2 + f_q \tilde{m}_u \left(\frac{R'}{z} \right)^{2c_u} \tilde{m}_u^\dagger f_q + f_q \tilde{m}_d \left(\frac{R'}{z} \right)^{2c_d} \tilde{m}_d^\dagger f_q \right] H_q U_{L,d}.
 \tag{A.9}$$

This time there are new sources of off-diagonal terms: the boundary masses \tilde{m} and the Hermitian rotations H .

B. SO(5) generators

B.1 Spinorial

The smallest SO(5) representation is the **4** spinorial. The generators in a convenient basis:

$$T_L^a = \frac{1}{2} \begin{bmatrix} \sigma^a & 0 \\ 0 & 0 \end{bmatrix} \qquad T_R^a = \frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & \sigma^a \end{bmatrix}
 \tag{B.1}$$

$$T_C^a = \frac{i}{2\sqrt{2}} \begin{bmatrix} 0 & \sigma^a \\ -\sigma^a & 0 \end{bmatrix} \qquad T_C^4 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}
 \tag{B.2}$$

T_L^a and T_R^a generate the $SO(4) \equiv SU(2)_L \times SU(2)_R$ subgroup of $SO(5)$ and $T_C^{\hat{a}}$ are the coset generators. The T_3 generators of the $SU(2)_L \times SU(2)_R$ subgroup are diagonal in this basis. This makes transparent how the $SU(2)_L \times SU(2)_R$ quantum numbers are embedded in $\mathbf{4}$:

$$\Psi = \begin{bmatrix} q_{+0} \\ q_{-0} \\ q_{0+} \\ q_{0-} \end{bmatrix} \quad \mathbf{4} = (\mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2}) \quad (\text{B.3})$$

where \pm stands for $\pm 1/2$. The Wilson-line exponential $e^{i\sqrt{2}hT_C^4}$ rotates $q_{\pm 0}$ into $q_{0\pm}$ and vice-versa:

$$\begin{aligned} q_{\pm 0} &\rightarrow \cos(h/2)q_{\pm 0} + i \sin(h/2)q_{0\pm} \\ q_{0\pm} &\rightarrow i \sin(h/2)q_{0\pm} + \cos(h/2)q_{\pm 0} \end{aligned} \quad (\text{B.4})$$

B.2 Fundamental

The 10 generators of the **fundamental representation** in an inconvenient basis:

$$\begin{aligned} T_{L,ij}^a &= -\frac{i}{2} \left[\frac{1}{2} \epsilon^{abc} (\delta_i^b \delta_j^c - \delta_j^b \delta_i^c) + (\delta_i^a \delta_j^4 - \delta_j^a \delta_i^4) \right] \quad a = 1 \dots 3 \\ T_{R,ij}^a &= -\frac{i}{2} \left[\frac{1}{2} \epsilon^{abc} (\delta_i^b \delta_j^c - \delta_j^b \delta_i^c) - (\delta_i^a \delta_j^4 - \delta_j^a \delta_i^4) \right] \quad a = 1 \dots 3 \\ T_{C,ij}^{\hat{a}} &= -\frac{i}{\sqrt{2}} \left[\delta_i^{\hat{a}} \delta_j^5 - \delta_j^{\hat{a}} \delta_i^5 \right] \quad \hat{a} = 1 \dots 4 \end{aligned} \quad (\text{B.5})$$

The generators are normalized as $\text{Tr} T^\alpha T^\beta = \delta^{\alpha\beta}$. The T_3 generators of $SU(2)_L \times SU(2)_R$ are non-diagonal in this basis. The quantum numbers of the 5-vector components can be found as:

$$Q = \frac{1}{\sqrt{2}} \begin{pmatrix} q_{++} + q_{--} \\ iq_{++} - iq_{--} \\ q_{+-} + q_{-+} \\ iq_{+-} - iq_{-+} \\ \sqrt{2}q_{00} \end{pmatrix} \quad \mathbf{5} = (\mathbf{2}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{1}) \quad (\text{B.6})$$

The Wilson-line transformation $\exp(i\sqrt{2}hT_C^4)$ rotates (q_{+-}, q_{-+}, q_{00}) into one another:

$$\begin{aligned} q_{+-} &\rightarrow \frac{1 + \cos(h)}{2} q_{+-} + \frac{1 - \cos(h)}{2} q_{-+} - i \frac{\sin(h)}{\sqrt{2}} q_{00} \\ q_{-+} &\rightarrow \frac{1 - \cos(h)}{2} q_{+-} + \frac{1 + \cos(h)}{2} q_{-+} + i \frac{\sin(h)}{\sqrt{2}} q_{00} \\ q_{00} &\rightarrow -i \frac{\sin(h)}{\sqrt{2}} q_{+-} + i \frac{\sin(h)}{\sqrt{2}} q_{-+} + \cos(h) q_{00} \end{aligned} \quad (\text{B.7})$$

B.3 Adjoint

The antisymmetric tensor Φ transforms as $\Phi \rightarrow \Omega \Phi \Omega^T$. We can represent the tensor as:

$$\Phi = \Phi_L^a T_L^a + \Phi_R^a T_R^a + \Phi_C^{\hat{a}} T_C^{\hat{a}} \quad (\text{B.8})$$

where T are $SO(5)$ generators in the fundamental representation. The $SO(5)$ commutation relations

$$[T_L^a, T_L^b] = i\epsilon^{abc}T_L^c \quad [T_R^a, T_R^b] = i\epsilon^{abc}T_R^c \quad [T_L^a, T_R^b] = 0 \quad (\text{B.9})$$

$$[T_C^a, T_C^b] = \frac{i}{2}\epsilon^{abc}(T_L^c + T_R^c) \quad [T_C^a, T_C^4] = \frac{i}{2}(T_L^a - T_R^a) \quad (\text{B.10})$$

$$[T_{L,R}^a, T_C^b] = \frac{i}{2}(\epsilon^{abc}T_C^c \pm \delta^{ab}T_C^4) \quad [T_{L,R}^a, T_C^4] = \mp \frac{i}{2}T_C^a \quad (\text{B.11})$$

imply that Φ_L is an $SU(2)_L$ triplet, Φ_R is an $SU(2)_R$ triplet and Φ_C is an $SU(2)_L \times SU(2)_R$ bifundamental, and the embedding of the quantum numbers is

$$(\mathbf{3}, \mathbf{1}) : \quad \Phi_L^1 \pm i\Phi_L^2 \rightarrow (\pm 1, 0) \quad \Phi_L^3 \rightarrow (0, 0) \quad (\text{B.12})$$

$$(\mathbf{1}, \mathbf{3}) : \quad \Phi_R^1 \pm i\Phi_R^2 \rightarrow (0, \pm 1) \quad \Phi_R^3 \rightarrow (0, 0) \quad (\text{B.13})$$

$$(\mathbf{2}, \mathbf{2}) : \quad \Phi_C^1 \pm i\Phi_C^2 \rightarrow (\pm 1/2, \pm 1/2) \quad \Phi_C^3 \pm i\Phi_C^4 \rightarrow (\pm 1/2, \mp 1/2) \quad (\text{B.14})$$

$$\mathbf{10} = (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}) \oplus (\mathbf{2}, \mathbf{2}) \quad (\text{B.15})$$

The Wilson-line transformation rotates the axial combination of the left and right triplets into the bifundamental

$$\begin{aligned} \frac{\Phi_L^a + \Phi_R^a}{\sqrt{2}} &\rightarrow \frac{\Phi_L^a + \Phi_R^a}{\sqrt{2}} \\ \frac{\Phi_L^a - \Phi_R^a}{\sqrt{2}} &\rightarrow \frac{\Phi_L^a - \Phi_R^a}{\sqrt{2}} \cos(h) + \Phi_C^a \sin(h) \\ \Phi_C^a &\rightarrow -\frac{\Phi_L^a - \Phi_R^a}{\sqrt{2}} \sin(h) + \Phi_C^a \cos(h) \end{aligned} \quad (\text{B.16})$$

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